ANNOUNCEMENT

INTRODUCTION TO REAL ANALYSIS

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http://ramanujan.math.trinity.edu/wtrench/index.shtml

A complete instructor's solution manual is available by email to wtrench@trinity.edu, subject to verification of the requestor's faculty status.

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Preface

This is a text for a two-term course in introductory real analysis for junior or senior mathematics majors and science students with a serious interest in mathematics. Prospective educators or mathematically gifted high school students can also benefit from the mathematical maturity that can be gained from an introductory real analysis course.

The book is designed to fill the gaps left in the development of calculus as it is usually presented in an elementary course, and to provide the background required for insight into more advanced courses in pure and applied mathematics. The standard elementary calculus sequence is the only specific prerequisite for Chapters 1–5, which deal with real-valued functions. (However, other analysis oriented courses, such as elementary differential equation, also provide useful preparatory experience.) Chapters 6 and 7 require a working knowledge of determinants, matrices and linear transformations, typically available from a first course in linear algebra. Chapter 8 is accessible after completion of Chapters 1–5.

Without taking a position for or against the current reforms in mathematics teaching, I think it is fair to say that the transition from elementary courses such as calculus, linear algebra, and differential equations to a rigorous real analysis course is a bigger step today than it was just a few years ago. To make this step today's students need more help than their predecessors did, and must be coached and encouraged more. Therefore, while striving throughout to maintain a high level of rigor, I have tried to write as clearly and informally as possible. In this connection I find it useful to address the student in the second person. I have included 295 completely worked out examples to illustrate and clarify all major theorems and definitions.

I have emphasized careful statements of definitions and theorems and have tried to be complete and detailed in proofs, except for omissions left to exercises. I give a thorough treatment of real-valued functions before considering vector-valued functions. In making the transition from one to several variables and from real-valued to vector-valued functions, I have left to the student some proofs that are essentially repetitions of earlier theorems. I believe that working through the details of straightforward generalizations of more elementary results is good practice for the student.

Great care has gone into the preparation of the 760 numbered exercises, many with multiple parts. They range from routine to very difficult. Hints are provided for the more difficult parts of the exercises.

Organization

Chapter 1 is concerned with the real number system. Section 1.1 begins with a brief discussion of the axioms for a complete ordered field, but no attempt is made to develop the reals from them; rather, it is assumed that the student is familiar with the consequences of these axioms, except for one: completeness. Since the difference between a rigorous and nonrigorous treatment of calculus can be described largely in terms of the attitude taken toward completeness, I have devoted considerable effort to developing its consequences. Section 1.2 is about induction. Although this may seem out of place in a real analysis course, I have found that the typical beginning real analysis student simply cannot do an induction proof without reviewing the method. Section 1.3 is devoted to elementary set theory and the topology of the real line, ending with the Heine-Borel and Bolzano-Weierstrass theorems.

Chapter 2 covers the differential calculus of functions of one variable: limits, continuity, differentiablility, L'Hospital's rule, and Taylor's theorem. The emphasis is on rigorous presentation of principles; no attempt is made to develop the properties of specific elementary functions. Even though this may not be done rigorously in most contemporary calculus courses, I believe that the student's time is better spent on principles rather than on reestablishing familiar formulas and relationships.

Chapter 3 is to devoted to the Riemann integral of functions of one variable. In Section 3.1 the integral is defined in the standard way in terms of Riemann sums. Upper and lower integrals are also defined there and used in Section 3.2 to study the existence of the integral. Section 3.3 is devoted to properties of the integral. Improper integrals are studied in Section 3.4. I believe that my treatment of improper integrals is more detailed than in most comparable textbooks. A more advanced look at the existence of the proper Riemann integral is given in Section 3.5, which concludes with Lebesgue's existence criterion. This section can be omitted without compromising the student's preparedness for subsequent sections.

Chapter 4 treats sequences and series. Sequences of constant are discussed in Section 4.1. I have chosen to make the concepts of limit inferior and limit superior parts of this development, mainly because this permits greater flexibility and generality, with little extra effort, in the study of infinite series. Section 4.2 provides a brief introduction to the way in which continuity and differentiability can be studied by means of sequences. Sections 4.3–4.5 treat infinite series of constant, sequences and infinite series of functions, and power series, again in greater detail than in most comparable textbooks. The instructor who chooses not to cover these sections completely can omit the less standard topics without loss in subsequent sections.

Chapter 5 is devoted to real-valued functions of several variables. It begins with a discussion of the toplogy of $\mathbb{R}^n$ in Section 5.1. Continuity and differentiability are discussed in Sections 5.2 and 5.3. The chain rule and Taylor's theorem are discussed in Section 5.4.

Chapter 6 covers the differential calculus of vector-valued functions of several variables. Section 6.1 reviews matrices, determinants, and linear transformations, which are integral parts of the differential calculus as presented here. In Section 6.2 the differential of a vector-valued function is defined as a linear transformation, and the chain rule is discussed in terms of composition of such functions. The inverse function theorem is the subject of Section 6.3, where the notion of branches of an inverse is introduced. In Section 6.4 the implicit function theorem is motivated by first considering linear transformations and then stated and proved in general.

Chapter 7 covers the integral calculus of real-valued functions of several variables. Multiple integrals are defined in Section 7.1, first over rectangular parallelepipeds and then over more general sets. The discussion deals with the multiple integral of a function whose discontinuities form a set of Jordan content zero. Section 7.2 deals with the evaluation by iterated integrals. Section 7.3 begins with the definition of Jordan measurability, followed by a derivation of the rule for change of content under a linear transformation, an intuitive formulation of the rule for change of variables in multiple integrals, and finally a careful statement and proof of the rule. The proof is complicated, but this is unavoidable.

Chapter 8 deals with metric spaces. The concept and properties of a metric space are introduced in Section 8.1. Section 8.2 discusses compactness in a metric space, and Section 8.3 discusses continuous functions on metric spaces.

Although this book has been published previously in hard copy, this electronic edition should be regarded as a first edition, since producing it involved the nontrivial task of combining LaTeX files that were originally submitted to the publisher separately, and introducing new fonts. Hence, there are undoubtedly errors–mathematical and typographical–in this edition. Corrections are welcome and will be incorporated when received.

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