MATH COLLOQUIUM

The (new) world of Gauss factorials

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Date : Wednesday, October 7, 2009
Time : 14:00
Place : TB 250, Boğaziçi Üniversitesi

Abstract: I will report on joint work with Karl Dilcher. Gauss's generalisation of Wilson's theorem was given in his renowned classic, the Disquisitiones Arithmeticae (1801). In [1] Karl Dilcher and I gave the first extension of the Gauss-Wilson theorem, the key new object of interest being what we call a 'Gauss factorial'. This entirely elementary object opens up a whole new world of interest, revealing many new results in Number Theory. By a pleasing co-incidence this new object has important consequences for another classic theorem of Gauss: his (1828) "beautiful (mod p) binomial coefficient congruence" (quote of Bruce Berndt and others), and an equally beautiful, closely related congruence of Jacobi (1837). The former concerns primes that are 1 mod 4, while the latter concerns primes that are 1 mod 3. In a 1983 Paris seminar Frits Beukers conjectured a mod p squared extension of Gauss's binomial coefficient congruence, and that conjecture was settled in 1986 by S. Chowla, B. Dwork and R. Evans. In the late 1980's, R. Evans and K. M. Yeung independently proved a mod p squared extension of Jacobi's congruence. No mod p cubed extension of either Gauss's or Jacobi's congruences had been conjectured, but last year – as a side outcome of another investigation of ours – we formulated and proved mod p cubed extensions of both the Gauss and Jacobi congruences [2]. In this entirely elementary talk - requiring no number theory background - I will introduce you to as much as possible of the above, and inform - if time allows - of some work-in-progress.

[1] John B. Cosgrave and Karl Dilcher, Extensions of the Gauss-Wilson theorem, Integers: Electronic Journal of Combinatorial Number Theory, 8, (2008)
[2] John B. Cosgrave and Karl Dilcher, Mod p^3 analogues of theorems of Gauss and Jacobi on binomial coefficients, Acta Arithmetica (to appear)

Tea and coffee will be served at 15:00