Differential Equations of Electrodynamics in Anisotropic Media: Computational Wave Simulation

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Abstract— The time-dependent electric and magnetic fields in homogeneous non-dispersive materials are governed by the following Maxwell's equations

$$\nabla \times \mathbf{H} = \varepsilon_0 \bar{\bar{\varepsilon}} \frac{\partial \mathbf{E}}{\partial t} + \mathbf{j}, \quad \nabla \times \mathbf{E} = -\mu_0 \bar{\bar{\mu}} \frac{\partial \mathbf{H}}{\partial t}, \qquad \nabla \cdot (\mu_0 \bar{\bar{\mu}} \mathbf{H}) = 0, \quad \nabla \cdot (\varepsilon_0 \bar{\bar{\varepsilon}} \mathbf{E}) = \rho, \tag{1}$$

where $x = (x_1, x_2, x_3)$ is a space variable from R^3, t is a time variable from R, $\mathbf{E} = (E_1, E_2, E_3)$, $\mathbf{H} = (H_1, H_2, H_3)$ are electric and magnetic fields, $E_k = E_k(x,t)$, $H_k = H_k(x,t)$, $k = 1,2,3$; $\mathbf{j} = (j_1, j_2, j_3)$ is the density of the electric current, $j_k = j_k(x,t)$, $k = 1,2,3$; ε_0 and μ_0 are the permittivity and permeability of the free space respectively, $\bar{\bar{\varepsilon}} = (\varepsilon_{ij})_{3\times 3}$, $\bar{\bar{\mu}} = (\mu_{ij})_{3\times 3}$ are 3×3 matrices; ρ is the density of electric charges. The electric charges and current are sources of electromagnetic waves. We assume these sources are given. It follows from (1) that electric charges and current have to satisfy the conservation law of charges

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0. \tag{2}$$

Here we suppose that

$$\mathbf{E} = 0, \quad \mathbf{H} = 0, \quad \mathbf{j} = 0, \quad \rho = 0, \quad \text{for} \quad t \leq 0. \tag{3}$$

The Cauchy problem for the system of crystal optics with smooth data and an exact solution of this problem has been described by Courant and Hilbert in [1] (see pages 603-612). Burridge and Qian in [2] have used a 'plane wave' approach to obtain an explicit formula for a fundamental solution of the same system of crystal optics. This formula has been expressed in terms of real loop integrals according to Herglotz-Petrovskii formula. Using this formula the numerical computation of the fundamental solution has been obtained and presented in the form of 1-D graphs. We note that the system of crystal optics has a great interest in mathematical physics and the different aspects of mathematical investigations for this system have been done in [3, 4, 5].

The most of electromagnetic scattering problems, initial value and initial boundary value problems have been solved by numerical methods, in particular, finite elements method, boundary elements methods, finite difference method, nodal method (see, for example, [6], [7], [8], [9]).

Nowadays computers can perform very complicated symbolic computations and this opens up new possibilities. Symbolic computations can be considered as useful tools for analytical methods that can provide exact solutions of problems. So, for example, Yakhno in [10] has used matrix symbolic computations for constructing an explicit formula for a fundamental solution (the Green's function) for the system of crystal optics. The approach of the paper [10] has been successfully used in [11, 12, 13, 14] for modeling and simulations of waves in different homogeneous anisotropic materials and media. The robustness of the suggested approach for the simulation of electric and magnetic waves is demonstrated by computed images of the electric and magnetic field components generated by a pulse dipole with a fixed polarization.

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