The approximation by the q-Bernstein polynomials

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For $q > 0$, $n \in \mathbf{Z}_+$, the *q-integer* $[n]_q$ and the *q-factorial* $[n]_q!$ are defined by

$$[n]_q := 1 + q + \ldots + q^{n-1} \quad \text{and} \quad [n]_q! := [1]_q[2]_q \ldots [n]_q,$$

respectively. For integers $0 \leq k \leq n$, the *q-binomial coefficient* is defined by

$$\begin{bmatrix} n \\ k \end{bmatrix}_q := \frac{[n]_q!}{[k]_q![n-k]_q!}\,.$$

Definition. Given $f : [0, 1] \to \mathbf{C}$, the q-Bernstein polynomials of f are:

$$B_{n,q}(f; z) := \sum_{k=0}^{n} f\left(\frac{[k]_q}{[n]_q}\right) z^k \prod_{j=0}^{n-1-k} \left(1 - q^j z\right), \quad n \in \mathbf{N}.$$

For $q = 1$, we recover the Bernstein polynomials, while for $q \neq 1$, we obtain new polynomials with rather different properties. In this talk, we discuss the possibilities of the uniform approximation of f by polynomials $B_{n,q}(f, .)$ both for $0 < q < 1$ and $q > 1$. The obtained results reveal some unexpected phenomena showing that the approximation properties of the q-Bernstein are essentially different from those of the Bernstein ones. Moreover, the cases $0 < q < 1$ and $q > 1$ are not similar to each other. The talk contains new results as well as those known previously.