Let $K$ $G_0$ be a semisimple linear algebraic group over an algebraically closed field $K$. Let $\rho : G_0 \to GL(V)$ be an irreducible representation of $G_0$, and let $G = K^* \cdot \rho(G_0)$ be the image of $G_0$ under $\rho$, adjoined by the scalar matrices. The $J-$irreducible monoid associated with the pair $(G_0, \rho)$ is the Zariski closure $M$ of the group $G$ in $End(V)$.

The group $G \times G$ acts on the monoid $M$ by $(g, h) \cdot x = gxh^{-1}$, $x \in M$, $(g, h) \in G \times G$. Let $J_x = GxG$ denote an orbit for some $x \in M$. There is a natural partial ordering on the set $\{J_x : \ x \in M\}$ of orbits:

$$J_e \leq J_f \iff J_e \subseteq \overline{J_f}, \ e, f \in M. \tag{1}$$

Here $\overline{J_f}$ denotes the Zariski closure of $J_f$ in $M$. The poset $\{J_x : \ x \in M\}$ is called the cross section lattice of $M$. Surprisingly, by a result of Putcha and Renner the cross section lattice of a $J-$irreducible monoid can be identified with a sublattice of the Boolean lattice of the set of nodes of the Dynkin diagram of $G_0$.

In this talk, we look into some combinatorial properties of the cross section lattice of a $J$-irreducible monoid associated with a semisimple group of one of the types $A_n$, $B_n$, or $C_n$. In particular, we determine which cross section lattices are supersolvable and compute their characteristic polynomials. If time permits we will talk about some combinatorial generalizations of the idea.