Semigroups of Operators Defined by Transition Probabilities

Abstract

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My goal in this series of lectures is to bring to your attention certain topics in the study of transition probabilities and transition functions (a transition function is a family of transition probabilities "glued" together by the Chapman-Kolmogorov equations). Essentially, I plan to outline an ergodic decomposition that stems from pioneering works of Kryloff, Bogoliouboff, Beboutoff, and Yosida, and, if time permits, to discuss briefly certain pointwise mean ergodic theorems.

To each transition probability P, we can associate in a very natural way two positive contractions S and T defined on a certain Banach lattice of measurable functions and a Banach lattice of measures, respectively. (All these notions will be defined rigorously during the lectures.) The ordered pair (S, T) is called a Markov pair. Generally, the results obtained for the sequences of iterates $(S^n)_{n\in\mathbb{N}}$ and $(T^n)_{n\in\mathbb{N}}$ have applications in any area in which transition probabilities are used. Thus, the results are applied in the study of discrete-time dynamical systems (like, for instance, in the study of the $\mathbb{Z}$-action defined by an element of $\mathrm{SL}(n, \mathbb{R})$ on the space of right cosets $(\mathrm{SL}(n, \mathbb{R})/M)_{\mathrm{R}}$, where M is a closed subgroup of $\mathrm{SL}(n, \mathbb{R})$; in most cases M is a lattice in $\mathrm{SL}(n, \mathbb{R})$), or in the study of discrete-time time-homogeneous Markov processes (as, for example, random walks, or processes defined by iterated function systems).

Given a transition function $(P_t)_{t\in[0,+\infty)}$ (where P_t, $t \geq 0$, are transition probabilities that satisfy the Chapman-Kolmogorov equations), we can consider the Markov pair (S_t, T_t) defined by P_t for every $t \in \mathbb{R}$, $t \geq 0$. It turns out that $(S_t)_{t\in[0,+\infty)}$ and $(T_t)_{t\in[0,+\infty)}$ are semigroups of operators, and their study is useful when dealing with flows and semiflows (many examples are flows defined on spaces of cosets), or in the study of continuous-time time-homogeneous Markov processes (for instance, interacting particle systems).

Our approach will be based on tools and concepts of functional analysis similar in spirit to the works of Şafak Alpay, Eduard Emel'yanov, their students, and their collaborators. However, we encourage everyone who is interested in Markov processes or spaces of cosets to attend. Also strongly encouraged to attend are people who are in the process of choosing an area of research; for them, I would like to mention that the topics that we will discuss are to a large extent "virgin territory" full of unexplored directions of research of various degrees of difficulty.