Triangular algebras and non-selfadjoint extensions of von Neumann algebras

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Kadison and Singer introduced the class of Triangular algebras in 1960, intitiating the systematic study of non-selfadjoint operator algebras. The prototype of a triangular algebra is the algebra of bounded operators, upper triangular with respect to a given orthonormal basis in a Hilbert space. They called a subalgebra $\mathfrak{B}$ of $\mathcal{B}(\mathcal{H})$ triangular if $\mathfrak{B} \cap (\mathfrak{B})^*$ is a maximal abelian self-adjoint subalgebra(masa in short) of $\mathcal{B}(\mathcal{H})$. I will begin the talk with a a survey of the theory of triangular algebras.

In a couple of recent papers in the PNAS, Ge and Yuan, seeking a more intimate connection between non-selfadjoint and self-adjoint algebras, introduced the class of Kadison-Singer algebras. Given a von Neumann algebra $\mathfrak{M} \subseteq B(H)$, a Kadison-Singer algebra $\mathfrak{A}$ is a maximal reflexive algebra with diagonal $\mathfrak{M}$, ie, $\mathfrak{A}^* \cap \mathfrak{A} = \mathfrak{M}$. Kadison-Singer algebras generalize the most interesting class of triangular algebras and also generalize the class of nest algebras, which have both been extensively studied. I will show how the study of these algebras throws up some tantalizing connections between the theories of non-selfadjoint and von Neumann algebras.

In this talk, I will construct a large family of examples of Kadison-Singer algebras, prove structure results and indicate connections to famous problems in the theory of von Neumann algebras.