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Control of Heat Processes: Theory and Numerics

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Abstract

The numerical approximation of exact or trajectory controls for the wave equation is known to be, since the pionnering work of Glowinski-Lions in the nineties, a delicate issue, because of the anomalous behavior of the high frequency spurious numerical waves. Various efficient remedies have been developed and analyzed in the last decade to filter out these high frequency components: Fourier filtering, Tychonoff regularization, mixed finite element methods, multi-grid strategies, etc. Recently convergence rates results have also been obtained.

In this talk we present our recent work in collaboration with A. Münch in the context of the heat equation, which is the opposite paradigm because of its strong dissipativity and smoothing properties. The existing analytical results guarantee that, at least in some simple situations as in the finite-difference scheme in 1-d, the null or trajectory controls for numerical approximation schemes converge. This is due to the intrinsic high frequency damping of the heat equation that is inherited by its numerical approximation schemes. But when developing numerical simulations the topic appears to be much more subtle and difficult. In fact, efficiently computing the null control for a numerical approximation scheme of the heat equation is a difficult problem in itself. The difficulty is strongly related to the ill-posedness of the backward heat problem.

The controls of minimal L^2-norm are characterized as minima of quadratic functionals on the solutions of the adjoint heat equation, or its numerical versions. These functionals are shown to be coercive in very large spaces of solutions, sufficient to guarantee the L^2 character of controls, but very far from being identifiable as energy spaces for the adjoint system. This very weak coercivity of the functionals under consideration makes the approximation problem to be exponentially ill-posed and the functional framework to be far from being well adapted to standard techniques in numerical analysis. In practice, the controls of minimal L^2-norm exhibit a singular highly oscillatory behavior near the final controllability time, which can not be captured numerically. Standard techniques such as Tychonoff regularization or quasi-reversibility methods allow to slightly smooth the singularities but reduce significantly the quality of the approximation.

We develop the numerical version of the so-called transmutation method that allows writing the control of a heat process in terms of the corresponding control of the associated wave process, by means of a "time convolution" with a one-dimensional *controlled fundamental heat solution*. This method, although it can be proved to converge, is also subtle in its computational implementation. Indeed, in one hand, it requires using convergent numerical schemes for the control of the wave equation, a problem that, as mentioned above, is delicate in itself. But it also needs computing an accurate approximation of a controlled fundamental heat solution, an issue that requires its own analysis and significant numerical and computational new developments.

These methods are thoroughly illustrated and discussed along the paper, accompanied by some numerical experiments in one space dimension that show the subtlety of the issue. These experiments allow comparing the efficiency of the various methods. This is done in the case where the control is distributed in some subdomain of the domain where the heat process evolves but similar results and numerical experiments could be derived for other cases such as the one in which the control acts on the boundary.

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