1D DIRAC OPERATORS WITH SPECIAL PERIODIC POTENTIALS

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This talk presents our joint recent results (2010) with Boris Mityagin (Ohio State University, USA).

For one-dimensional Dirac operators of the form

$$Ly = i\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\frac{dy}{dx} + vy, \quad v = \begin{pmatrix} 0 & P \\ Q & 0 \end{pmatrix}, \ \ y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

we single out a class X of π-periodic potentials v with the following properties:

(i) The smoothness of potentials v is determined only by the rate of decay of related spectral gaps $\gamma_n = |\lambda_n^+ - \lambda_n^-|$, where $\lambda_n^\pm$ are the eigenvalues of $L = L(v)$ considered on $[0, \pi]$ with periodic (for even n) or antiperiodic (for odd n) boundary conditions.

(ii) There is a Riesz basis in $L^2([0, \pi], \mathbb{C}^2)$ which consists of periodic (or antiperiodic) eigenfunctions and associated functions (at most finitely many).

In particular, the class X contains the families of symmetric potentials X_{sym} (defined by $\overline{Q} = P$) and skew-symmetric potentials $X_{skew-sym}$ (defined by $\overline{Q} = -P$), or more generally the families $X_t, \ t \in \mathbb{R}\backslash\{0\}$, defined by $\overline{Q} = tP$. Finite-zone potentials belonging to X_t are dense in X_t.