

ON THE PAPER “AN ABSTRACT VERSION OF THE KOROVKIN APPROXIMATION THEOREM”

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ABSTRACT. We claim that the proof of the main result of the paper [1] is incorrect.

Following the terminology of the paper [1], we claim that the proof of Lemma 3.4 is incorrect, as follows: Inequality (3.14) states that, for all $x \in X$,

$$|L_n(h_x; x)| \leq \varepsilon L_n(1; x) + \frac{M}{m} |L_n(P_x; x)|.$$

Then taking supremum over $x \in X$, in [1] it is stated that

$$\max_{x \in X} |L_n(h_x; x)| \leq \varepsilon \|L_n\| + \frac{M}{m} \max_{x \in X} |L_n(P_x; x)|.$$

But it is not true. Indeed, although m and M are depending on $x \in X$, the authors of the paper ignore this! Letting m_x and M_x instead of m and M , the above inequality must be in the following form:

$$|L_n(h_x; x)| \leq \varepsilon L_n(1; x) + \frac{M_x}{m_x} |L_n(P_x; x)|.$$

Then taking supremum over $x \in X$ we get

$$\sup_{x \in X} |L_n(h_x; x)| \leq \varepsilon L_n(1; x) + \sup_{x \in X} \left(\frac{M_x}{m_x} |L_n(P_x; x)| \right).$$

But no guarantees that

$$\sup_{x \in X} \left(\frac{M_x}{m_x} |L_n(P_x; x)| \right) < \infty.$$

Indeed, it is possible that

$$\sup_{x \in X} M_x = \infty$$

or

$$\sup_{x \in X} \frac{1}{m_x} = \infty.$$

Hence the proof of the Lemma 3.4 is incorrect. Since this lemma is used to prove Theorem 3.1 (main result of the paper), the proof of the main result of the paper [1] is incorrect.

This paper has been cited in many other papers, so I believe that the readers should be warned.

REFERENCES

- [1] O. Duman & C. Orhan, *An abstract version of the Korovkin approximation theorem*, Publ. Math. Debrecen 69/1-2 (2006), 33-46187-197.

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