Matchings in graphs, circuits and toric ideals

Anargyros Katsampekis

Toric ideals are binomial ideals which represent the algebraic relations of finite sets of power products. They have applications in diverse areas in mathematics, such as algebraic statistics, integer programming, hypergeometric differential equations, graph theory, etc.

A basic problem in Commutative Algebra asks one to compute the least number of polynomials needed to generate the toric ideal up to radical. This number is commonly known as the arithmetical rank of a toric ideal. A usual approach to this problem is to restrict to a certain class of polynomials and ask how many polynomials from this class can generate the toric ideal up to radical. Restricting the polynomials to the class of binomials we arrive at the notion of the binomial arithmetical rank of a toric ideal.

In the talk we study the binomial arithmetical rank of the toric ideal I_G of a finite graph G in two cases:

(1) G is bipartite,

(2) I_G is generated by quadratic binomials.

Using a generalized notion of a matching in a graph and circuits of toric ideals, we prove that, in both cases, the binomial arithmetical rank equals the minimal number of generators of I_G.

Department of Mathematics
Mimar Sinan Fine Arts University
34380 Istanbul
Turkey