Some average results connected with reductions of groups of rational numbers

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Abstract

Let $\Gamma \subset \mathbb{Q}^*$ be a finitely generated subgroup and let p be a prime number such that the reduction group Γ_p is a well defined subgroup of the multiplicative group $\mathbb{F}_p^*$. In the first part of the talk, given that $\Gamma \subseteq \mathbb{Q}^*$, assuming the Generalized Riemann Hypothesis, we determine an asymptotic formula for the average over prime numbers, powers of the order of the reduction group Γ_p. The case of rank 1, when Γ is generated by only one rational number, was previously considered by C. Pomerance and P. Kurlberg. In the second part of the talk, for any $m \in \mathbb{N}$ we determine an asymptotic formula for the average of the number of primes $p \leq x$ for which the index $[\mathbb{F}_p^* : \Gamma_p] = m$. The average is performed over all finitely generated subgroups $\Gamma = \langle a_1, \ldots, a_r \rangle \subset \mathbb{Q}^*$, with $a_i \in \mathbb{Z}$ and $a_i \leq T_i$ with a range of uniformity: $T_i > \exp(4(\log x \log\log x)^{\frac{1}{2}})$ for every $i = 1, \ldots, r$. The case of rank 1 and $m = 1$ corresponds to the classical Artin conjecture for primitive roots and has already been considered by P. J. Stephens in 1969. The second part of the talk is a joint work with Lorenzo Menici.