Representations of Leavitt path algebras

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(Joint work with Murad Özaydın)

Abstract

LPAs were defined recently (Abrams and Aranda Pino, 2005; Ara, Moreno and Pardo, 2007) but they have roots in the works of Leavitt in the 60s focused on understanding the extent of the failure of the IBN (Invariant Basis Number) property for arbitrary rings. A ring has IBN if any two bases of a finitely generated free module have the same number of elements. Fields, division rings, commutative rings, Noetherian rings all have IBN. However, the rings L(1,n) defined by Leavitt (1962) and their analytic cousins the C*-algebras of Cuntz (1977) are not artificial and pathological structures constructed only for the sake of providing counter examples; for instance, they implicitly come up in signal processing (as the algebras generated by the downsampling and upsampling operators). Moreover Leavitt's work (1962, 1965) provided important impetus for major developments in non-commutative ring theory in the 1970s by Cohn, Bergman and others.

In joint work with Murad Özaydın, we study the (unital) representations of a Leavitt path algebra $L(\Gamma)$ of a di(rected)graph Γ over a field. We show that the category of $L(\Gamma)$-modules is equivalent to a subcategory of quiver representations. We give a necessary and sufficient criterion for the existence of a nonzero finite dimensional representation. When Γ is a row-finite digraph we determine all possible finite dimensional quotients of $L(\Gamma)$ and we classify all finite dimensional $L(\Gamma)$-modules via an explicit Morita equivalence given by an effective (reduction) algorithm on Γ.

References

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