Inverse source problems and approximation methods for heat equations

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Abstract.

Heat source identification problems are the most commonly encountered inverse problems in heat conduction. These problems have been studied over several decades due to their significance not only in a variety of scientific and engineering applications, but also to their significance in the theory of inverse source problems for PDEs.

In this talk we consider following inverse problems of identifying the unknown time-dependent heat source $H(t)$ or spacewise dependent heat source $F(x)$ of the heat conduction problem

$$\begin{cases} u_t = (k(x)u_x)_x + F(x)H(t), \quad (x,t) \in \Omega_T, \\ u(x,0) = u_0(x), \quad x \in (0,l), \\ u_x(0,t) = 0, \quad u(l,t) = 0, \quad t \in (0,T_f], \end{cases} \tag{1}$$

with the separable sources of the form $F(x)H(t)$, from supplementary temperature measurements

$$\begin{cases} M_1 := u_T(x) = u(x,T_f), \quad x \in (0,l) \ \text{(measured final data);} \\ M_2 := h(t) = u(0,t), \quad t \in (0,T_f] \ \text{(measured temperature on the left end of a rod)} \\ M_3 := f(t) = -k(l)u_x(l,t), \quad x \in (0,l) \ \text{(measured flux at the right end of a rod)} \\ M_4 := U_T(x) = \int_0^{T_f} u(x,t)dt, \quad x \in (0,l) \ \text{(nonlocal or integral type observations).} \end{cases} \tag{2}$$

Here $\Omega_T := \{(x,t) \in \mathbb{R}^2 : x \in (0,l),\ t \in (0,T_f],\ \ T_f > 0\}$ is the parabolic domain. The all data in (2) are defined to be the *measured output data*. Note that the measured output data may has a noise.

We are concerned with the following *two types of inverse source problems*. In the *first type inverse problems*, defined to be as ISPF, the *spacewise source term $F(x)$ needs to be recovered from one of the measured output data (2), assuming that the function $H(t)$ is known.* In the *second type inverse problems*, defined to be as ISPH, the *time-depended source term $H(t)$ needs to be recovered from one of the measured output data (2), assuming that the function $F(x)$ is known.* For a given source terms $F(x)$ and $H(t)$ the problem in (1) is defined to be the *Direct Problem* or *Forward Problem*.

Two main approaches for the inverse problems ISPF and ISPH can be employed. The first approach is based on the "Congugate Gradient Method (CGM)" and uses an explicit formulas for the gradients of the cost functionals $J(F) = \|u(F) - M_i\|^2_{L_2}$ and $J(H) = \|u(H) - M_i\|^2_{L_2}$ corresponding to the above defined problems ISPF and ISPH, via the corresponding adjoint problem solutions. Based on this approach and explicit gradient formulas for the functionals $J(F)$ and $J(H)$ are derived and the implemented for numerical solution of the problems ISPF and ISPH. The second approach uses the "Forward Collocation Method (FCM)". The algorithm of FCM is also permits one to estimate the degree of ill-posedness of the problems ISPF and ISPH. Alternatively, Fourier method can be employed to illustrate the comparison of spacewise ($F(x)$) and time-dependent ($H(t)$) heat source identification problems.

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