A polynomial embedding of pairs of orthogonal partial latin squares

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In this talk we will first give general information and main results concerning Latin squares. Then we will present the new embedding result on orthogonal partial Latin squares.

Let N represent a set of n distinct elements. A non-empty subset P of $N \times N \times N$ is said to be a *partial latin square*, of order n, if for all $(x_1, x_2, x_3), (y_1, y_2, y_3) \in P$ and for all distinct $i, j, k \in \{1, 2, 3\}$,

$$x_i = y_i \text{ and } x_j = y_j \text{ implies } x_k = y_k.$$

If $|P| = n^2$, then we say that P is a *latin square*, of order n.

Two partial latin squares P and Q, of the same order are said to be *orthogonal* if they have the same non-empty cells and for all $r_1, c_1, r_2, c_2, x, y \in N$

$$\{(r_1, c_1, x), (r_2, c_2, x)\} \subseteq P \text{ implies } \{(r_1, c_1, y), (r_2, c_2, y)\} \nsubseteq Q.$$

In 1960 Evans proved that a partial latin square of order n can always be embedded in some latin square of order t for every $t \geqslant 2n$. In the same paper Evans raised the question as to whether a pair of finite partial latin squares which are orthogonal can be embedded in a pair of finite orthogonal latin squares. We show that a pair of orthogonal partial latin squares of order t can be embedded in a pair of orthogonal latin squares of order at most $16t^4$ and all orders greater than or equal to $48t^4$. This is the first polynomial embedding result of its kind.