Boğaziçi MATH COLLOQUIUM

Sets of Spectral Synthesis: A Short Survey and Some Recent Results

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Abstract: Consider the Lebesgue space $L^1(\mathbb{R}^n)$ on the Euclidean group $(\mathbb{R}^n, +)$. Equipped with its usual norm and the convolution as the product, $L^1(\mathbb{R}^n)$ is a normed ring. If I is any closed ideal of $L^1(\mathbb{R}^n)$, the closed set $E = \bigcap_{f \in I} (\widehat{f})^{-1}(0)$ is the hull of the ideal. Here, \widehat{f} is the Fourier transform of f. One of the most important problems about the ring $L^1(\mathbb{R}^n)$ is to describe its closed ideals in terms of their hulls. This is however an impossible problem. The difficulty lies in the fact that in general there are infinitely many distinct closed ideals with the same hull. A closed subset E of \mathbb{R}^n is said to be a set of synthesis if the ideal $k(E) = \{f \in L^1(\mathbb{R}^n) : \widehat{f} = 0 \text{ on } E\}$ is the only closed ideal with hull E. In the first part of the talk I will present a survey of the known results about sets of synthesis. In the second part I will discuss some recent results obtained jointly with E. Kaniuth about synthesibility of the closed sets E on which there is a group of isomorphisms of \mathbb{R}^n acting on E transitively.

This talk is a part of the Workshop on Banach Algebras organized by İstanbul Center for Mathematical Sciences (IMBM).

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