

# İSTANBUL ANALYSIS SEMINARS

## CROSS RATIO PROBLEM ON SOME SUBCLASSES OF UNIVALENT FUNCTIONS

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**Abstract:** Let  $D$  be a simply connected domain in the closed complex plane  $\mathbb{C}$ . For given distinct points  $z_k$  ( $k = 1, \dots, 4$ ) in  $D$ , let  $(z_1, z_2, z_3, z_4)$  denote their cross ratio. It is well-known that if  $w = h(z)$  is a Möbius transformation and  $w_k = h(z_k)$ , then  $(z_1, z_2, z_3, z_4) = (w_1, w_2, w_3, w_4)$  but it may not be so if  $h$  is an arbitrary univalent function in  $D$ . If  $w = f(z)$  is univalent in  $D$ , then the quotient  $(w_1, w_2, w_3, w_4)/(z_1, z_2, z_3, z_4)$ , denoted by  $Q(f)$ , determines a **measure of deviation** of  $f$  from a Möbius transformation. So there arises the problem of finding the variability region of  $Q(f)$  when  $f$  belongs to a class of univalent functions which is compact in the space  $H(D)$  of holomorphic functions in  $D$ .

In this talk we will point out a couple of results in this direction and give a concrete result for a small class “univalent second degree polynomials.”

## References

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- [2] Y. Avci & E. Złotkiewicz, “An extremal problem for univalent functions,” *İstanbul Üniv. Fen Fak. Mat. Derg.* **50** (1991), 159–164 (1993).
- [3] Y. Avci & E. Złotkiewicz, “A few remarks on convex mappings,” *Math. Balkanica (N.S.)* **13** (1999), no. 1-2, 47–54.
- [4] James A. Jenkins, “The method of the extremal metric,” in: *Handbook of Complex Analysis: Geometric Function Theory, Vol. 1*, R. Kühnau (ed.), pp. 393–456. Elsevier Science B.V., North-Holland, Amsterdam, 2002.

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