Degenerate separable differential operators and applications

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ABSTRACT

The nonlocal boundary value problems(BVPs) for degenerate differential-operator equations with variable coefficients are studied. The L_p separability properties of elliptic problems and well-posedeness of parabolic problems in mixed $L_{\mathbf{p}}$ spaces are derived. Then by using the regularity properties of linear problems, the existence and uniqueness of solution of nonlinear elliptic problem is obtained. Note that applications of these problems can be models of different physics process.

The main objective of the present paper is to discuss maximal regularity properties of the following degenerate elliptic equation

$$-\varepsilon a\left(x\right)u^{[2]}\left(x\right)+A\left(x\right)u\left(x\right)+\varepsilon^{\frac{1}{2}}A_1\left(x\right)u^{[1]}\left(x\right)+A_0\left(x\right)u\left(x\right)+\lambda u=f\left(x\right), \quad (0.1)$$

where

$$D_x^{[i]}u=u^{[i]}\left(x\right)=\left(x^{\gamma}\frac{d}{dx}\right)^{i}u\left(x\right),\ \gamma\geq 0,\ x\in\left(0,1\right),$$

ε is a small positive parameter, λ is a complex parameter, $a\left(x\right)$ is a complex valued function and A, A_0, A_1 are linear operators in a Banach space E. We prove that this problem is isomorphism from $W_{p,\gamma}^{[2]}\left(0,1;E\left(A\right),E\right)$ onto $L_p\left(0,1;E\right)\times E_1\times E_2$, where E_k are interpolation spaces between $E\left(A\right)$ and E (see section 1 for definition of these spaces). In section 3, we show that the problem (0.1) is $L_p\left(0,1;E\right)$ separabile, i.e., we prove that for $f\in L_p\left(0,1;E\right)$ the problem (0.1) has a unique solution $u\in W_{p,\gamma}^{[2]}\left(0,1;E\left(A\right),E\right)$ and the uniform coercive estimate holds

$$\sum_{i=0}^{2}\left|\lambda\right|^{1-\frac{i}{2}}\varepsilon^{\frac{i}{2}}\left\|u^{[i]}\right\|_{L_p(0,1;E)}+\left\|Au\right\|_{L_p(0,1;E)}\leq C\left\|f\right\|_{L_p(0,1;E)}$$

for $\left|\arg\lambda\right|\leq\varphi$, $\varphi<\pi$ with sufficiently large $\left|\lambda\right|$, where the constant C depend only on p and A.

The uniform well-posedeness of initial and BVP for the degenerate abstract parabolic equation

$$\frac{\partial u}{\partial t}+\varepsilon a\left(x\right)\frac{\partial^{[2]}u}{\partial x^2}+A\left(x\right)u\left(x,t\right)=f\left(x,t\right)$$

is established in E-valued mixed $L_{\mathbf{p}}$ space.

Then we derive the existence and uniqueness of nonlocal BVP for the following nonlinear degenerate abstract equation

$$-q\left(x\right)u^{[2]}\left(x\right)+B\left(x,u,u^{(1)}\right)u\left(x\right)=F\left(x,u,u^{(1)}\right),$$

where q is a real valued function, B and F are nonlinear operator in a Banach space E.

Finally, we give the application of the abstract problem to general BVPs, particularly Wentzell BVP for PDE.