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MULTINORMED VON NEUMANN ALGEBRAS OF TYPE I

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Abstract: The present talk is devoted to classification of multinormed (or locally convex) von Neumann algebras of type I. A multinormed von Neumann algebra is defined as an inverse limit of von Neumann algebras whose connecting maps are w^*-continuous $*$-homomorphisms. It is known that the bounded part of a multinormed von Neumann algebra is a von Neumann algebra, and every multinormed von Neumann algebra is a central completion $\mathcal{M}_{\mathcal{E}}$ of a von Neumann algebra $\mathcal{M}$ equipped with a domain $\mathcal{E}$ of its central projections. Moreover, $\mathcal{M}_{\mathcal{E}}$ admits the predual $(\mathcal{M}_{\mathcal{E}})_*$ which is an ℓ^1-normed space equipped with the canonical bornology $\{\operatorname{ball}\mathcal{M}_* e : e \in \mathcal{E}\}$. We prove that $(\mathcal{M}_{\mathcal{E}})_* = \mathcal{M}_{*\mathcal{E}} = \sum_{e\in\mathcal{E}} \mathcal{M}_* e = \mathcal{M}_* \otimes_{\mathcal{E}} \mathcal{E}_*$ and the bornological dual $(\mathcal{M}_{*\mathcal{E}})'$ is identified with $\mathcal{M}_{\mathcal{E}}$ up to an isometric isomorphism of polynormed spaces. In the case of $L^\infty(\mathcal{T})$, the domain $\mathcal{E}$ corresponds to a measurable covering of a locally compact space equipped with a positive Radon integral $\int : C_c(\mathcal{T}) \to \mathbb{C}$, and the algebra $L^\infty(\mathcal{T})_{\mathcal{E}}$ is represented by means of $\mathcal{E}$-locally essentially bounded functions, that is, those functions $f \in \mathfrak{L}(\mathcal{T})$ such that $\operatorname{esssup}|f|E| < \infty$ for all $E \in \mathcal{E}$. The bornological predual $L^\infty(\mathcal{T})_{*\mathcal{E}}$ of the multinormed von Neumann algebra $L^\infty(\mathcal{T})_{\mathcal{E}}$ is reduced to the ℓ^1-normed space $L^1(\mathcal{T})_{\mathcal{E}}$ which consists of those $g \in L^1(\mathcal{T})$ such that $\|g\|_1 = \int_E |g|$ for some $E \in \mathcal{E}$. If the σ-covering is reduced to the trivial one $\mathcal{E} = (\mathcal{T})$ then $L^\infty(\mathcal{T})_{\mathcal{E}} = L^\infty(\mathcal{T})$, $L^1(\mathcal{T})_{\mathcal{E}} = L^1(\mathcal{T})$ and we obtain the classical result $L^1(\mathcal{T})^* = L^\infty(\mathcal{T})$. In the case of $L^\infty(\mathcal{T})\overline{\otimes}\mathcal{M}$ with a measurable covering $\mathcal{E}$ of $\mathcal{T}$ and a (multiplicity) von Neumann algebra $\mathcal{M} \subseteq \mathcal{B}(H)$, we obtain the multinormed von Neumann algebra $L^\infty(\mathcal{T})_{\mathcal{E}}\overline{\otimes}\mathcal{M}$, which consists of unbounded decomposable operators $\int^{\oplus} x(t)$ on their common domain $\mathcal{O} = \cup_{E\in\mathcal{E}}([E]\otimes 1)(L^2_H(\mathcal{T}))$ defined by means of (unbounded) measurable functions $x(\cdot) : \mathcal{T} \to \mathcal{M}$ which are $\mathcal{E}$-locally bounded in the sense that all functions $([E]x)(\cdot) : \mathcal{T} \to \mathcal{M}$, $e \in \mathcal{E}$ are bounded. Moreover, $(L^\infty(\mathcal{T})_{\mathcal{E}}\overline{\otimes}\mathcal{M})_{*\mathcal{E}\otimes 1} = L^1(\mathcal{T})_{\mathcal{E}} \otimes_{\ell^1} \mathcal{M}_*$, which consists of those $y \in L^1_{\mathcal{M}_*}(\mathcal{T})$ such that $\|y\|_1 = \int_E \|y(t)\|$ for some $E \in \mathcal{E}$. Finally, every multinormed von Neumann algebra of type I can be obtained by means of $L^\infty(\mathcal{T})_{\mathcal{E}}\overline{\otimes}\mathcal{M}$ for various multiplicities $\mathcal{M}$.

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