Algebra, Geometry and Topology of Singularities
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Galatasaray University-Istanbul
Abstracts

Bobadilla's Conjecture

David Massey

Recently, Bobadilla has made a conjecture about the non-splitting of the Milnor fiber cohomology along an irreducible 1-dimensional critical locus of a hypersurface singularity. This conjecture is related to the long-standing conjecture of Lê on singular surface germs. We will reformulate Bobadilla's topological conjecture and turn it into an algebra problem. And then we will discuss special cases for which the conjecture is true.

Fujita's freeness conjecture for 5-fold

Zhixian Zhu

Let X be a smooth projective variety of dimension n and L any ample line bundle. Fujita conjectured that the adjoint line bundle $O(K_X + mL)$ is globally generated for any m greater or equal to $n+1$. By studying the singularity of pairs, we will prove Fujita's freeness conjecture for smooth 5-folds.

Jet schemes and resolution of singularities

Hussein Mourtada

I will talk on the one hand about the notion of a generating sequence of a valuation, and on the other hand about the relation between jet schemes and divisorial valuations. I will then describe how this relation allows one to construct generating sequences of some divisorial valuations; this provides a constructive approach to a conjecture of Teissier on resolution of singularities with toric morphisms.

On the ideal defining the higher Nash blowup of a hypersurface.

Andrés Daniel Duarte

In this talk we will discuss some known results on blowing ups of modules in the particular case of the higher Nash blowup. We will describe an explicit ideal whose blowing up defines the higher Nash blowup of a hypersurface. Using this ideal we will prove a higher-order version of Nobile's theorem for normal hypersurfaces.

Ultrametric spaces of branches on arborescent singularities

Patrick Popescu-Pampu

I will present joint work with Evelia García Barroso and Pedro González Pérez. We say that a normal complex analytic surface singularity is *arborescent* if the dual graph of any resolution of it is a tree. I will explain that any branch on such a singularity determines a canonical ultrametric on the set of branches different from it and that this ultrametric encodes topological information about the structure of the embedded resolutions of any finite set of branches. This generalizes theorems of Płoski and Favre-Jonsson, which concern the case when both the arborescent singularity and the branch on it are smooth.

Topology of spaces of valuations and geometry of singularities

Ana Belén de Felipe Paramio

Given an algebraic variety X defined over a field k, the space of all valuations of the field of rational functions of X extending the trivial valuation on k is a projective limit of algebraic varieties. This space had an important role in the program of Zariski for the proof of the existence of resolution of singularities.

In this talk we will consider the subspace $RZ(X, x)$ consisting of those valuations which are centered in a given closed point x of X and we will focus on the topology of this space. In particular we will concentrate on the relation between its homeomorphism type and the local geometry of X at x. We will characterize this homeomorphism type for regular points and normal surface singularities. This will be done by studying the relation between $RZ(X, x)$ and the normalized non-Archimedean link of x in X coming from the point of view of Berkovich geometry.

Sets with few rational points

Georges Comte

We explain how to prove that some categories of sets contain few rational points (of bounded height). The sets we are going to investigate are subanalytic p-adic, or even definable sets in some more general non archimedean context, as well as real sets slowly oscillating.

Lipschitz normal embeddings and Determinantal Singularities

Helge Møller Pedersen

An algebraic singularity X has two natural metrics. Both are defined using an embedding to Euclidian space, but are independent of the embedding up to bilipschitz equivalence. The first is the outer metric given by restricting the Euclidian metric to X. The other is the inner metric, where the distance between two points are defined as the infimum of the lengths of curves in X between the points. It is clear that the inner distance between two points is equal or larger that their outer distance. The other way is in general not true, and one says that X is Lipschitz normally embedded if there exist a constant K, such that the inner distance is less than or equal to the K times the outer distance. This poster will discuss the case of determinantal singularities. We will show that the model (or generic) determinantal singularity, that is the set of matrices of rank less than a given number, is Lipschitz normally embedded. We will also discuss the case of when a general determinantal singularity is Lipschitz normally embedded.

Primitive cohomology of smooth projective complete and non-complete intersections

Javier Fernandez de Bobadilla

Work in progress.

Let X be a smooth projective manifold. Let $F_1, ..., F_r$ be a set of homogeneous polynomials, all of them of the same degree (sufficiently high) defining X as a subscheme of the projective space. Let c be the codimension of X. Pick $G_1, ..., G_c$, c generic linear combinations of $F_1, ..., F_r$. The complete intersection $Z_0 := V(G_1, ..., G_c)$ contains X as an irreducible component. Let

$$Z_t := V(G_{1,t}, ..., G_{c,t})$$

be a 1-parameter smoothing of Z_0. Our aim is to compare the intermediate primitive cohomology of X (for a certain polarisation) with the intermediate cohomology of Z_t.

If $\dim(X) \leq 3$ we find a natural embedding of the intermediate primitive cohomology of X into the intermediate cohomology of Z_t. For $\dim(X) \geq 4$ this embedding does not exist in general. We find a necessary and sufficient condition: we define two polynomials P and Q on the Chern classes of the tangent bundle of X and on the polarisation given by the embedding, and the cohomology embedding holds if and only if Q is a multiple of P in the cohomology ring of X.

The condition is satisfied inmediately if X is a complete intersection, but also if the codimension of X is sufficiently low. This may be seen as a supporting evidence of Hartshorne conjecture on smooth varieties of small codimension being complete intersections, and perhaps as a tool to address it.

In the case that the condition is not satisfied, the piece of the primitive cohomology that does not embed is shown to embed into the intermediate primitive cohomology of a manifold of dimension 4 less that the original one, setting up in this way an inductive scheme to analyze it.

Relative enumerative invariants of real rational surfaces

Ilia Itenberg

The purpose of the talk is to present real analogs of relative Gromov-Witten invariants in several situations. For example, for real del Pezzo surfaces with a real (-2)-curve, we suggest, under some assumptions, an invariant signed count of real rational curves that belong to a given divisor class and are tangent to the (-2)-curve at each intersection point; the resulting number does not depend neither on the point constrains, nor on deformation of the surface preserving the real structure and the (-2)-curve. (Joint work with V. Kharlamov and E. Shustin.)

Symplectic fillings of lens spaces as Lefschetz fibration

Burak Özbağcı

We construct a positive allowable Lefschetz fibration over the disk on any minimal (weak) symplectic filling of the canonical contact structure on a lens space. Using this construction we prove that any minimal symplectic filling of the canonical contact structure on a lens space is obtained by a sequence of rational blowdowns from the minimal resolution of the corresponding complex two-dimensional cyclic quotient singularity. This is a joint work with Mohan Bhupal.

Arithmetics of curvettes on quotient surface singularities

José Ignacio Cogolludo

Given a normal surface X the generalized Riemann-Roch formula ([7, 11, 3, 2, 1])

$$\chi(\mathcal{O}_X(D)) = \chi(X) + D \cdot (D - K_X) + R_X(D)$$

allows one to relate the Euler characteristics of $\mathcal{O}_X(D)$ and $\mathcal{O}_X$ via the canonical divisor K_X and a correcting term $R_X(D)$. Such a term is the main object of this talk. The invariant $R_X : Cl(X) \to \mathbb{Q}$ only depends on the rational divisor class in X, that is, the quotient of the Weil group by Cartier divisors and can be defined as a sum of associated invariants at the singular points of X ([4]). For this reason we only consider the local case. Assume $X = \mathbb{C}^n/G$, the group $Cl(X)$ is naturally isomorphic to the group of characters of G, that is, $G^\vee = Hom(G, \mathbb{C}^*)$. Following M.Reid's notation [12] we will write $\mathcal{O}(a)$ for the rational divisor class in X associated with the character $a \in G^\vee$. In other words,

$$\text{if } D_1, D_2 \in \mathcal{O}(a) \text{ then } R_X(D_1) = R_X(D_2) =: R_X(a).$$

Consider a resolution π of the singularity X. A curvette in X is a $\mathbb{Q}$-divisor whose strict transform by π is smooth, irreducible, and it intersects the excepcional divisor transversally. A *generic divisor* can be naturally defined as a $\mathbb{Q}$-divisor which factors as curvettes whose preimages by π are disjoint. We will show that not every rational divisor class contains curvettes, in particular their generic divisors might not be irreducible. We will describe generic divisors and will obtain formulas for RX using this description.

In the case of cyclic quotient singularities $X = \frac{1}{d}(1, p)$, this description can be given via the Hirzebruch-Jung continued fraction descomposition $[q_1, ..., q_n]$ of $\frac{p}{d}$ and the use of the greedy algorithm ([10, 9]) of $a \in \mathbb{Z}_d = G^\vee$ with respect to $[q_1, ..., q_n]$. Possible applications of these results are lattice point counting formulas for rational polytopes [5] and Kouchnirenko's ([8, 6]) formulas for the Milnor number of a curve on a normal surface singularity.

Kapranov's Theorem and Quantifier Elimination

Gönenç Onay

Abstract: After proving Kapranov's Theorem (one of the central result in Tropical Geometry) I will show that it is essentially equivalent to the fact that the theory of algebraically closed valued fields admits quantifier elimination. If time permits I will discuss further parallelism in the case of valued fields with extra structure (e.g. equipped with an automorphism/a derivation) or in non algebraically closed case (in which Kapranov's theorem is not valid).

Efficient Computation of Dual Space and Directional Multiplicity of an Isolated Singularity

Zafeirakis Zafeirakopoulos

The information characterizing an isolated singularity can be captured in a local dual basis, expressing combinations of vanishing derivatives at the singular point. Macaulay's algorithm is a classic algorithm for computing such a basis, for a point in an algebraic set. The integration method of Mourrain constructs much smaller matrices than Macaulay's approach, by performing integration on previously computed elements. In this work we are interested in the efficiency of dual basis computation, as well as its relation to orthogonal projection. First, we introduce an easy to implement criterion that avoids redundant computations during the computation of the dual basis, by deleting certain columns from the matrices in the integration method. Second, we introduce the notion of directional multiplicity, which expresses the multiplicity structure with respect to an axis, and is useful in understanding the geometry behind projection. We use this notion to shed light on the gap between the degree of the generator of the elimination ideal and the corresponding factor in the resultant.

Singular Implicit and inverse function theorem

Jaroslaw Wlodarczyk

Building upon ideas of Hironaka, Bierstone-Milman, Malgrange and others we generalize the inverse and implicit function theorem (in differential, analytic and algebraic setting) to sets of functions of larger multiplicities (or ideals). This allows one to describe singularities given by a finite set of generators or by ideals in a simpler form. In the special Cohen-Macaulay case we obtain a singular analog of the inverse function theorem. The technique leads to a canonical reduction of the strong Hironaka desingularization with normally flat centers to a so called resolution of marked ideals.

On a smooth quartic surface containing 56 lines

Ichiro Shimada

Simple Singular Irreducible Plane Sextics

Aysegul Ozguner Akyol

We consider complex plane projective curves of degree six with simple singular points only and classify such curves up to equisingular deformation. (We concentrate on the so-called non-special curves, as the special ones are already known). We list all sets of singularities realized by such curves, discuss their relation to the maximizing sets (i.e., those of total Milnor number 19), and, for each set of singularities found, describe the connected components of the moduli space. We also discuss the question of the realizability of a given set of singularities by a real curve.

Algebraic-tropical correspondence

Ilya Tyompki

In recent years tropical geometry became a powerful tool in enumerative algebraic geometry. In some cases it allows one to reduce the algebraic problem to a combinatorial problem by replacing the algebraic varieties with appropriate piece-wise linear objects and proving the "correspondence" theorem. In my talk I will describe the algebraic-tropical correspondence for rational curves in toric varieties satisfying toric and cross-ratio constraints.

Invariants of Gorenstein Surface singularities

José Seade

We will discuss invariants of isolated, normal, Gorenstaein surface singularities.