Boğaziçi Mathematics

Research Seminar

Vanishing Theorems on a compact complex manifold and applications to the Hopf Conjecture and the existence of complex structure on the 6-sphere

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Abstract: In [1], we prove a Weitzenbock formula on complex manifolds, which involves the Hodge Laplacian Δ_H, the Bochner Laplacian of the Levi-Civita connection Δ_R, and another Laplacian we construct, Δ_K, that is related to the Lefschetz operator and ∂ operator such that for any $(p,q)-$form, $\Delta_K + \Delta_H - 2\Delta_R = F(R) +$"quadratic terms" where $F(R) : E^{p,q} \to E^{p,q}$ is a curvature operator [7], [9]. This generalizes a Weitzenbock formula of Wu on Kähler manifolds such that $\Delta_H - \Delta_R = F(R)$ in [11]. Under certain conditions, we show that this Weitzenbock formula provides vanishing theorems for the Dolbeault cohomology groups of complex differential $(p,q)-$forms and obtain information about the Hodge numbers of the manifold, $h^{p,q}$, in particular, the geometric genus, $\mathfrak{p}_g$, and arithmetic genus, $\mathfrak{p}_a$, and irregularity, $\mathfrak{q}$, of a compact complex manifold under certain conditions [6], [2]. Furthermore, we use the main vanishing theorem to obtain the Euler characteristic of the manifold $\chi(M)$ to show that the Hopf Conjecture [8] holds for a compact complex manifold with nonnegative sectional, holomorphic bisectional and isotropic curvature under certain extra conditions for $(p,q)-$forms [10], [3]. Finally, an earlier result of Alfred Gray states that a hypothetical integrable almost complex structure on the 6-sphere, S^6, has to satisfy $h^{0,1}\left(S^6\right) > 0$ [5]. We apply our main vanishing theorem in [4] for $(0,1)-$forms to show that $h^{0,1}\left(S^6\right) = 0$ and thus, under certain additional conditions S^6 can not admit an integrable almost complex structure.

Date : Wednesday, December 07, 2016

Time: 15:00

Place: TB 250, Boğaziçi University

References

[1] Ferahlar, C. *A Weitzenbock Formula for Compact Complex Manifolds and Applications to the Hopf Conjecture in Real Dimension 6*, Thesis Dissertation, Lehigh University, 2016, 168 pages; Proquest, publication number: 10125873

[2] Ferahlar, C. *Vanishing Theorems and Plurigenera of a compact complex manifold.* Preprint. 2016.

[3] Ferahlar, C. *On Certain Compact Complex manifolds and the Hopf Conjecture.* Preprint. 2016.

[4] Ferahlar, C., Johnson, D. L. *Existence of an hypothetical integrable almost complex structure on 6-sphere.* Preprint. 2016.

[5] Gray, A. *A Property of a Hypothetical complex structure on the six sphere.* Bolletino U.M.I., 1997, **3**, 251–255, MR1456264

[6] Hirzebruch, F. *Topological methods in algebraic geometry,* Third ed. Die Grundlehren der Mathematischen Wissenschaften, Band 131, Springer-Verlag New York, Inc., 1966. MR 0202713

[7] Hodge, W. V. D., *The Theory and Applications of Harmonic Integrals.* Cambridge University Press, 1941, MR 0003947.

[8] H. Hopf. *Differentialgeometrie und topologische Gestalt.* Jahresber. Deutsch. Math.-Verein, 41: 209–229, 1932.

[9] Voisin, C. *Hodge theory and complex algebraic geometry. I,* Cambridge Studies in Advanced Mathematics, **Vol. 76**, English Ed. , Cambridge University Press, Cambridge, (2009j:32014). MR2451566

[10] Wells, Jr., R. O., *Differential analysis on complex manifolds.* Graduate Texts in Mathematics, **Vol. 65**, 2nd Ed. Springer-Verlag, New York-Berlin, (1980). MR608414

[11] Wu, H. H. *The Bochner technique in differential geometry,* Math. Rep., Mathematical Reports, **Vol. 3**, 1988, 289–538, MR1079031