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Research note

Mehmet Nadir: An amateur mathematician in Ottoman Turkey

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Abstract

This note summarizes the results of a recent survey of all the mathematical work of Mehmet Nadir, a Turkish amateur mathematician and professional educator who lived from 1856 to 1927 during the last years of the Ottoman Empire and the first years of the Turkish Republic. It is shown that, although working in isolated and adverse conditions, Nadir was able to establish a continuous correspondence with mathematicians in western Europe and, through his studies in number theory, obtained some results of lasting value.

Özet

Bu yazıda önce Osmanlı imparatorluğunun son döneminde, 1856–1927 yılları arasında yaşamış olan Mehmet Nadir'in yaşamı ve eğitime katkıları kısaca özetleniyor. Sonra matematik alanındaki çalışmaları tümüyle gözden geçiriliyor ve kalıcı değeri olan bir kaç buluşu ayrıntılı biçimde anlatılıyor. Böylece matematik tarihinde özgün bir yer tutmaya hak kazandığı iddia ediliyor.

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1. Introduction

It is well known that the new era of development in mathematics that started in central and western Europe after the Renaissance came to the Ottoman realm rather late. The first courses on differential and integral calculus were given at the military engineering school established in Istanbul in 1795. The

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educational effort in the 19th century was mostly concentrated in teaching, based on the translation of western textbooks, with no attempt to do any research. The first Ph.D. granted to a Turkish mathematician was the one obtained by Kerim Erim in 1919 at the Friedrich–Alexander University of Erlangen, with his thesis "Über die Trägheitsformen eines Modulsystems."

The present research was started with the aim of finding out whether any Turkish mathematicians published research results before the period of the first doctoral degrees. We have scanned the *Jahrbuch der Fortschritte der Mathematik* from 1868 to 1914 and the *Revue semestrielle des Publications mathématiques* from 1893 to 1914 and found only three Turkish names: Tevfik Pasha, Salih Zeki, and Mehmet Nadir. The *Jahrbuch* mentions Tevfik Pasha's book "Linear Algebra" (in English, second edition published in 1893) and Salih Zeki's article "Notation algébrique chez les Orientaux" published in the *Journal Asiatique* in 1898, while Mehmet Nadir's contributions to *l'Intermédiaire des Mathématiciens* are mentioned once in the *Jahrbuch* and 24 times in the *Revue semestrielle*.

The appearance of Mehmet Nadir in the abstract journals surprised us. Until the present research was carried out he was not a figure mentioned in the reviews of the development of mathematics in Ottoman Turkey. On the other hand, an inquiry among some of his former students showed that he was much admired by those who believed that he had obtained original results in number theory.

It was clear that the only way to reach an adequate assessment was to make a complete survey of his publications and look for any positive reactions from the world of mathematics. Such a search was carried out by the present author and the results were reported in a book in Turkish, published in 1997 [İnönü, 1997]. Here a brief summary is given.

2. Sources

The main source of information for a biography of Mehmet Nadir is a section in the book "The History of Education in Turkey," written by Osman Ergin in Turkish, first published in 1941 and reprinted in 1977 [Ergin, 1941, vol. 3, 814–827]. In addition to this source, I have benefited from other written reminiscences by students of Nadir and by people who knew him, from private conversations with two granddaughters, Mrs. Leyla Tekeli and Mrs. Lale Göksu, from the studies of Professor Enginün on the history of literature, and from the private archives of Taha Toros.

3. Life story

Mehmet Nadir was born on the island of Chios, a short distance away from the Anatolian mainland, in the second half of the 19th century. We have no documents for the year of his birth, but it may be estimated to be 1856 or 1857. His parents were quite poor people. At an early age he was adopted by the captain of a ferry boat serving Chios, brought to Turkey, and placed in a military school in Bursa. He turned out to be a brilliant student with a special aptitude for mathematics. He continued his education at the famed Kuleli military school in Istanbul and finally graduated from the Naval Academy as a Navy staff lieutenant.

In recognition of his merits, he was appointed as secretary to the Chamber of the Council of the first Admiralty (Bahriye Meclisi Başkanlığı Özel Kalemi), which looked like the beginning of a successful career in the Ottoman navy. However, his independent character did not go well with the discipline of the military bureaucracy. In less than a year, he was transferred to the Naval Academy as an instructor for all the mathematics courses. He was also given additional teaching duty at the Darüşşafaka High School, the prestigious semistate institution. At these two schools, Nadir quickly acquired the reputation of a learned and exacting teacher.

Now follows a period of 2 years, crucial for the future course of his life, on which we have very scant information. It is only known that in the beginning of 1879, Nadir abandoned his duties at both schools, went abroad to Britain, attended some night courses in London, and came back after 2 years, at which time he was imprisoned aboard the battleship Mesudiye for neglect of duty. He was released 1 year later, being discharged from the navy for good.

With this discharge, the doors of officialdom were closed to Nadir. He turned to private education and in this area he became amazingly successful. The school he founded in 1884 under the name of Nümunei Terakki ("Example of Progress") became the most popular and progressive institution in Istanbul for primary and secondary education for boys and girls. At the height of its development, it housed about 600 students and the best teachers in the city. In addition to his duties as director, Nadir wrote several articles for various newspapers on modern theories of education, on developments in science, and on literary subjects. He was the first person who published translations of passages from Shakespeare's plays into Turkish [Enginün, 1971].

Nadir's successful directorship lasted for 13 years. It was terminated abruptly in 1897 when a plot for dethroning Sultan Abdulhamid, organized by the Istanbul branch of the revolutionary committee, "Union and Progress," in which some teachers of the school, including Mehmet Nadir, were involved, was uncovered at the last minute through an indiscretion of Nadir himself [Hanioğlu, 1986, p. 187]. Nümunei Terakki was taken over by the Ministry of Education and Nadir was transferred to the directorship of the Aşiret School, a special school run by the Palace for the sons of tribal leaders from various parts of the Empire. Separated from his accustomed milieu, Nadir turned to his first love, mathematics. His first contributions to the French journal, *l'Intermédiaire des Mathématiciens*, appeared during this period, in 1900 and 1901.

In 1902, after a revolt of the students at the Aşiret School, Nadir was sent to Aleppo in Syria, in charge of the local educational administration. As he had become persona non grata for both the Abdulhamid regime and its successor, his exile continued after the change of power in Istanbul in 1908. From Aleppo he was sent to Tripoli in Libya and then to Edirne in Thrace. Finally, in 1912, he was allowed to return to the capital.

In Istanbul, he had difficulty at first in finding a job, but eventually his former students at the Ministry of Education obtained grace from the authorities. First, in 1915, he was appointed as a lecturer to the newly instituted University for Women (İnas Darülfünunu). Then, in 1919, a chair for number theory was established at Istanbul University (called Darülfünun at that time), which Nadir occupied until his death in 1927.

4. Nadir's publications in mathematics

The mathematical studies of Mehmet Nadir deal almost exclusively with number theory and more particularly with Diophantine equations. They have been published in three places: the French journal *l'Intermédiaire des mathématiciens*, the periodical of the Istanbul Faculty of Sciences, and Nadir's textbook on number theory.

4.1. Communications in l'Intermédiaire des Mathématiciens

This journal was started in 1894 by two graduates of the École Polytechnique, C.A. Laisant (Ph.D. in science) and E. Lemoine (civil engineer). In their first issue the editors explained their aim of serving people of all categories interested in mathematics by informing them about research subjects and by providing solutions to proposed problems [Laisant and Lemoine, 1894]. Its policy was to publish all problems sent to the journal, irrespective of whether they were of an elementary character or not, and then in subsequent issues publish the solutions provided by other readers. The journal was well received, in particular by amateur mathematicians, and continued its existence for several decades into the 20th century.

During the years 1900–1914 Nadir sent to this journal 26 questions and 36 answers. When the journal received more than one answer, the editors, as a rule, published one of them, mentioning only the names of the other contributors. In this vein, 12 answers by Nadir were published. All the contributions by Nadir can be found in [İnönü, 1997]. Here I shall reproduce only three of the answers, as they appear to be more interesting than the others. Two of them are mentioned in L.E. Dickson's encyclopedic treatise, "History of the Theory of Numbers" [Dickson, 1919–1920], and the third is the solution of a problem that remained unsolved for about 10 years.

4.1.1. Question by Arcitenens (a pseudonym of an unknown correspondent) in vol. 14, 1902, p. 244

Can you indicate integral number solutions to the equation

$$x^2 + y^2 - z^2 = u^5$$

other than the particular solution $x = 3$, $y = 12$, $z = 11$, $u = 2$?

Answer given by Mehmet Nadir in vol. 15, 1908, pp. 46–47:

(a) First I have found the following formulae which give an infinite number of integral solutions to the (slightly different) equation:

$$x^2 + y^2 + z^2 = u^5$$

as follows:

$$\begin{aligned} x &= a(a^2 + b^2)^2 \\ y &= b(a^2 + b^2)(a^2 - b^2) \\ z &= 2ab^2(a^2 + b^2) \\ u &= a^2 + b^2, \end{aligned}$$

where a and b are arbitrary integers.

(b) Using these relations I have derived the following expressions which give an infinite number of integral solutions to the proposed equation $y^2 + v^2 - w^2 = u^5$, as follows:

$$y = b(a^2 + b^2)(a^2 - b^2)$$

$$v = \frac{1}{2}\lfloor (a^2+1)(a^2+b^2)^2 + 4b^4 \rfloor$$
$$w = \frac{1}{2}\lfloor (a^2-1)(a^2+b^2)^2 - 4b^4 \rfloor$$
$$u = a^2 + b^2,$$

where b is an arbitrary integer, but a must be an odd integer to obtain integral solutions.

As an example, Nadir indicates the case of $a = 1$, $b = 2$, which gives in (a) $25^2 + 30^2 + 40^2 = 5^5$ and in (b) $30^2 + 57^2 - 32^2 = 5^5$.

These solutions of Nadir are mentioned by Dickson in the second volume on Diophantine analysis [Dickson, 1919–1920, vol. 2, ch. XX, p. 544].

4.1.2. Question by Mehmet Nadir in vol. 18, 1911, p. 217, which is mentioned in the Revue semestrielle

For the equations,

$$\frac{x^4 + y^4 + z^4}{2} = u^2 + v^2 + w^2 = p^2,$$

I have found the following two sets of integral solutions:

$$x = 6,\ y = 3,\ z = 9,\ u = 27,\ v = 54,\ w = 18,\ p = 63.$$
$$x = 11,\ y = 5,\ z = 16,\ u = 80,\ v = 176,\ w = 55,\ p = 201.$$

Are there general formulae which give an infinite number of sets of integral solutions?

This equation together with the two special solutions is again mentioned in Dickson's book [Dickson, 1919–1920, vol. 2, ch. XXII, p. 659], giving reference to Mehmet Nadir's contribution in *l'Intermédiaire des mathématiciens*. Dickson indicates also that general solutions were obtained by E. Miot and A. Cunningham using the identity

$$x^4 + y^4 + (x+y)^4 = 2\left(x^2 + xy + y^2\right)^2$$

given in vol. 19, 1912, p. 70 of the same journal.

Let me note in passing that actually only the set of E. Miot,

$$x = m, \quad y = n, \quad z = m + n; \qquad u = m^2 + mn, \quad v = n^2 + mn, \quad w = mn; \qquad p = m^2 + mn + n^2$$

solves the equation proposed by Nadir, while the set given by A. Cunningham as

$$x = B - A, \quad y = B + A, \quad z = 2B,$$
$$u = 3B^2 - A^2 + 2AB, \quad v = -3B^2 + A^2 + 2AB, \quad w = 4AB,$$
$$p = A^2 + 3B^2$$

is a solution of the related equation

$$x^4 + y^4 + z^4 = u^2 + v^2 + w^2 = 2p^2.$$

4.1.3. Question by A. Boutin, first asked in 1897 and then again in 1906, as no answer came in the meantime

Can one find solutions valid for any positive n, to the following congruence:

$$a^{\alpha n+\beta} + b^{\alpha' n+\beta'} + c^{\alpha'' n+\beta''} + d^{\alpha''' n+\beta'''} \equiv 0 \pmod{p},$$

where a, b, c, d, p are different prime numbers, p being the largest; the α's are positive integers, none of which is zero, while the β's are integers which may be positive, negative or zero. It is also required that this congruence should not be divisible into two pieces as:

$$a^{\alpha n+\beta} + b^{\alpha' n+\beta'} \equiv 0 \quad \text{and} \quad c^{\alpha'' n+\beta''} + d^{\alpha''' n+\beta'''} \equiv 0 \pmod{p}.$$

The answer was given by Mehmet Nadir in vol. 15, 1908, p. 224 with the following special solutions:

$$\begin{aligned}
&7^{26+4} + 13^{13n+2} + 17^{26n+12} + 23^{4n+2} \equiv 0 \pmod{53},\\
&2^{5n+3} + 5^{3n+1} + 11^{30n+2} + 13^{30n+9} \equiv 0 \pmod{31},\\
&2^{9n+4} + 7^{24n+6} + 19^{36n+5} + 43^{34n+12} \equiv 0 \pmod{73}, \quad \text{and}\\
&2^{36n+10} + 3^{27n+8} + 5^{27n+6} + 7^{27n+1} \equiv 0 \pmod{109}.
\end{aligned}$$

In his communication, Nadir did not explain how he obtained these results. He gave this explanation later, in an article which appeared in 1925 in the *Journal of the Faculty of Sciences of Istanbul University*, pp. 178–183, as follows:

First, he wrote each term as $(a^\alpha)^n \cdot a^\beta$. Reminding us of Fermat's theorem

$$a^{p-1} \equiv 1 \pmod{p},$$

he points out that, to begin with, the congruences

$$a^\alpha \equiv 1 \pmod{p}, \quad b^{\alpha'} \equiv 1 \pmod{p}, \quad c^{\alpha''} \equiv 1 \pmod{p}, \quad d^{\alpha'''} \equiv 1 \pmod{p} \tag{1}$$

may be solved, and independently, one may find the βs that would satisfy the congruence:

$$a^\beta + b^{\beta'} + c^{\beta''} + d^{\beta'''} \equiv 0 \pmod{p}. \tag{2}$$

As an example, he chooses arbitrarily the set

$$p = 53, \quad a = 7, \quad b = 13, \quad c = 17, \quad d = 23$$

and selects one solution to each of the congruences (1) as

$$\alpha = 26 \quad \text{for } 7^\alpha \equiv 1 \pmod{53},$$

$$\alpha' = 13 \quad \text{for } 13^{\alpha'} \equiv 1 \pmod{53},$$
$$\alpha'' = 26 \quad \text{for } 17^{\alpha''} \equiv 1 \pmod{53}, \quad \text{and}$$
$$\alpha''' = 4 \quad \text{for } 23^{\alpha'''} \equiv 1 \pmod{53}.$$

Finally, he finds the set $\beta = 4$, $\beta^1 = 2$, $\beta'' = 12$, $\beta''' = 2$, which satisfies the congruence (2) and yields the result

$$7^{26n+4} + 13^{13n+2} + 17^{26n+12} + 23^{4n+2} \equiv 0 \pmod{53}.$$

4.2. A general algorithm for division

Mehmet Nadir first explained this algorithm in an article in the *Journal of the Faculty of Sciences of Istanbul University* of 1917, pp. 509–529. Later he described it in the first volume of his textbook on number theory, published in 1926. Unfortunately he did not publish it in any foreign journal and consequently this work was not noticed in the West. It is not mentioned in Dickson's book.

Nadir's purpose is to find a general method for obtaining the remainder in a division operation. He observes that the remainder r in the division of the number N by the number d will be the smallest positive number r satisfying the congruence

$$N \equiv r \pmod{d}.$$

If N were a number close to d, e.g., a number less than $2d$, we would immediately see what r should be. In fact, for any given N and d, one can subtract from N adequate multiples of d to bring the congruence into that easily solvable form.

To obtain an algorithm that would decrease N step by step, Nadir proceeds in the following way: let N be written by means of digits $a, b, \ldots, u$, with respect to any basis B, as

$$N = ab\ldots u.$$

By separating the last digit u and denoting the remaining digits by D, we can write N as

$$N = BD + u$$

and consequently

$$BD + u \equiv r \pmod{d}. \tag{3}$$

Now, choosing the numbers m and R properly, we can satisfy the following congruence:

$$Bm + R \equiv 0 \pmod{d}. \tag{4}$$

Combining Eqs. (3) and (4), we obtain the new congruence

$$RD - um \equiv -mr \pmod{d},$$

where the left-hand side is an integer smaller than N, since R is smaller than B.

One can repeat this step n times to arrive at the equation

$$P \equiv (-m)^n r \pmod d,$$

where P is a number smaller than d. This is the essence of Nadir's method. He designs an algorithm that carries out this step-by-step operation automatically.

By introducing various simplifications into his final equation, he is able to derive many of the well-known rules for division by specific numbers such as 7, 11, and 13, and he expresses his belief that some new rules may also be obtained by this method. His detailed results can be found in [İnönü, 1997, pp. 25–28].

4.3. Articles in the Journal of the Faculty of Sciences of Istanbul University (Darülfünun Fünun (Fen) Fakültesi Mecmuası)

Between the years 1916 and 1927, Nadir published 12 articles in this journal, all in Turkish. A complete list can be found in [İnönü, 1997, pp. 95–96]. Summaries of these articles are given in a publication by Feza Günergun [1995], who made a survey of all articles published in this journal during the years 1916–1933. It can be seen from this survey that most of the articles were of a didactic nature and did not involve research. Following this practice, Nadir's articles mostly give the basic notions of number theory. Only two of his articles, which I have already discussed, report the results of his own research.

4.3.1. Textbook on number theory

When, at the end of his career, Nadir was appointed to the number theory chair of Istanbul University, he started to write a textbook on number theory in Turkish. He was able to complete only the first volume [Nadir, 1926], which deals with the classical concepts of number theory and contains also his general algorithm for division.

5. An evaluation

My survey of the mathematical studies of Mehmet Nadir makes him appear essentially as an amateur mathematician. He did not have any academic training in the mathematics of his day. He had to spend most of his younger years in the area of education, working as a school director and teacher. When, in his fifties, he returned to mathematics, he could only deal with the elementary problems in number theory. It may be thought that such a figure does not deserve a special place in the history of mathematics. My reason for believing the contrary is based on his devotion to research and to his determination to try to find something new in spite of his insufficient knowledge and all the adverse circumstances of his life.

His inadequate training is a general feature among the people interested in science at that time in the Ottoman Empire. The other two Turkish mathematicians mentioned in the *Fortschritte der Mathematik* in the last part of the 19th century did not have degrees in mathematics, either, although they benefited from better circumstances. Tevfik Pasha spent several years in France and in the United States on official duty, during which times he was able to carry on private studies in current mathematics, while Salih Zeki received a degree in electrical engineering in Paris and joined the mathematics department of Istanbul

University a few years after his return from France. Tevfik Pasha and Salih Zeki knew much more mathematics than Mehmet Nadir. Tevfik Pasha attempted unsuccessfully to create a new algebra to compete with quaternions. Salih Zeki wrote textbooks in several fields of current mathematics, but carried out extensive research only in the history of mathematics. Compared with the work of these contemporaries, Nadir's studies appear to be elementary. But he alone, because of his dogged determination to achieve something new, was able to enter into continuous correspondence with other mathematicians in western Europe and keep it going for many years and was rewarded in the end by having two of his elementary solutions inserted into Dickson's book.

Mehmet Nadir is the first Turkish mathematician whose name appears as a contributor in a book written in modern times by an internationally known mathematician. This distinction gives him a special place in the history of Turkish mathematics. I believe he also earns a place in the general history of mathematics as one of the members of a devoted group of mathematicians (amateur or professional) who, by their contributions sent from all over the world (many of which found a place in Dickson's book), kept alive for so many years the *sui generis* journal, *l'Intermédiaire des mathématiciens*.

I should add that the publication of my book on Mehmet Nadir seems to have led to a reappraisal of his status in Turkey. In the newly published "History of Mathematical Literature during the Ottoman Period" [İhsanoğlu et al., 1999] there is a section devoted to his accomplishments in mathematics and education.

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