The geometry of finite non-associate division algebras.

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Finite non-associative division algebras (also called *semifields*) were first studied by L. E. Dickson in 1905 as axiomatically defined algebraic structures satisfying almost all of the axioms in the definition of a finite field. A study which naturally arose in the aftermath of the well-known theorem: *A finite skew-field is a field.* This theorem essentially says that the axiom of commutativity of multiplication is implied by the other axioms of a finite field.

Dickson showed that this does not hold true for the axiom of associativity of multiplication, by constructing explicit examples of finite non-associative division algebras. Later, when the coordinatisation of projective planes was established (1940's), it turned out that Dickson's examples also implied the existence of projective planes in which Desargues configuration does not hold, a configuration of points and lines, whose importance emerged from Hilbert's axiomatisation of geometry (1899).

Once this connection between the theory of semifields and the theory of projective planes was established, the topic received a considerable amount of attention from both geometers and algebraists. A good survey of the state of the art at that time, can be found in the book *Projective planes* (1970) by Hughes and Piper, the book *Finite Geometries* (1968) by Dembowski, or in Knuth's dissertation *Finite semifields and finite projective planes* (1963).

In the last decade a second wave of interest in the theory of semifields arrived, partly due to applications, partly due to new connections between the algebra and the geometry of semifields, e.g. [1], [2]. For a survey, see [3]. In this talk we will elaborate on the geometry of finite semifields, and explain how the interplay between algebra and geometry has allowed us to obtain many new results, e.g. [4, 5], including the classification of 8-dimensional rank two commutative semifields.

References

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