OSCILLATIONS OF DIFFERENTIAL AND DIFFERENCE EQUATIONS WITH SEVERAL DEVIATING ARGUMENTS

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ABSTRACT

Consider the first-order delay differential equation

$$x'(t) + \sum_{i=1}^{m} p_i(t)x(\tau_i(t)) = 0, \quad t \geq 0,$$

where, for every $i \in \{1, \ldots, m\}$, p_i is a continuous real-valued function in the interval $[0, \infty)$, and τ_i is a continuous real-valued function on $[0, \infty)$ such that

$$\tau_i(t) \leq t, \quad t \geq 0, \quad \text{and} \quad \lim_{t \to \infty} \tau_i(t) = \infty$$

and the discrete analogue difference equation

$$\Delta x(n) + \sum_{i=1}^{m} p_i(n)x(\tau_i(n)) = 0, \quad n \in \mathbb{N}_0,$$

where $\mathbb{N} \ni m \geq 2$, p_i, $1 \leq i \leq m$, are real sequences and $\{\tau_i(n)\}_{n \in \mathbb{N}_0}$, $1 \leq i \leq m$, are sequences of integers such that

$$\tau_i(n) \leq n - 1, \quad n \in \mathbb{N}_0, \quad \text{and} \quad \lim_{n \to \infty} \tau_i(n) = \infty, \quad 1 \leq i \leq m$$

Several optimal sufficient oscillation conditions for the above equations are presented.