

Piatetski Shapiro meets Chebotarev

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In 1953 Ilya Piatetski-Shapiro proved an analogue of the prime number theorem for primes of the form $[n^c]$, where $[x] = \max\{n \in \mathbb{N} : n \leq x\}$, n runs through positive integers and $c > 0$ is fixed. He showed that the number $\pi_c(x)$ of these primes (to be denoted by PS primes) not exceeding a given number x is asymptotic to $x^{1/c}/\log x$ provided that $c \in (1, 12/11)$. Since then, the admissible range of c has been extended by many authors and the result is currently known for $c \in (1, 2817/2426)$. In this talk, we give an asymptotic formula for PS primes lying in a specified Chebotarev class. Proving such an analogue of Chebotarev's density theorem gives rise to many interesting subsets of PS primes. For example one may show that there are infinitely many PS primes of the form $x^2 + ny^2$ for a range of c that depends only on n .