On Slant Geometry

Handan Yıldırım

Istanbul University

It is known that a geodesic in the Poincaré disk is a circle which is perpendicular to the ideal boundary (i.e., unit circle). If we adopt geodesics as lines in the Poincaré disk, we have the model of Hyperbolic geometry. We have another class of curves in the Poincaré disk which has an analogous property with lines in the Euclidean plane. A *horocycle* is a circle which is tangent to the ideal boundary. We note that a line in the Euclidean plane can be considered as the limit of the circles when the radius tends to infinity. In the same manner, a horocycle is also a curve which can be considered as the limit of the circles in the Poincaré disk when the radius tends to infinity. Hence, horocycles are also an analogous notion of lines. If we adopt horocycles as lines, what kind of geometry do we obtain? We say that two horocycles are *parallel* if they have common tangent point at the ideal boundary. Under this definition, the parallel axiom is satisfied. However, for any two fixed points in the Poincaré disk, there exist always two horocycles passing through these points, so that the first axiom of Euclidean geometry is not satisfied. In the case of general dimensions, this geometry is said to be *horospherical geometry.* On the other hand, we have another kind of curves in the Poincaré disk which has similar properties with Euclidean lines. An *equidistant curve* is a circle whose intersection with the ideal boundary consists of two points. Generally, the angle between an equidistant curve and the ideal boundary is $\phi \in (0, \pi/2]$. Here, we emphasize that a geodesic is an equidistant curve with $\phi = \pi/2$. However, a horocycle is not an equidistant curve, but it is a circle with $\phi = 0$. We call the geometry where $\phi = \pi/2$ *vertical geometry* and the geometry where $\phi = 0$ *horizontal geometry.* And also we call the family of geometry depending on ϕ *slant geometry.*

In this talk, I will give some of recent results about this geometry including joint works with Mikuri Asayama, Shyuichi Izumiya and Aiko Tamaoki.