ISTANBUL KÜLTÜR UNIVERSITY

Penrose tilings of the plane and noncommutative algebraic geometry

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Abstract

The space X of Penrose tilings of the plane has a natural topology on it. Two tilings are equivalent if one can be obtained from the other by an isometry. The quotient topological space X/~ is bad: every point in it is dense. The doctrine of non-commutative geometry is to refrain from passing to the quotient and construct a non-commutative algebra that encodes some of the data lost in passing to X/~. In this example (see Connes book for details) the relevant non-commutative algebra is a direct limit of products of matrix algebras. We will obtain this non-commutative algebra by starting with a certain quotient of the free algebra on two variables treated as the homogeneous coordinate ring of a non-commutative curve. This is similar to treating the preprojective algebra of a wild hereditary algebra as the homogeneous coordinate ring of a non-commutative curve.

The category of quasi-coherent sheaves on this non-commutative curve is equivalent to the module category over a simple von Neumann regular ring. That von Neumann regular ring is the same as the direct limit algebra that Connes associates to X/~. We will discuss algebraic analogues of various topological features of X/~. For example, the non-vanishing of extension groups between simple modules is analogous to the fact that every point in X/~ is dense (which is equivalent that any finite region of one Penrose tiling appears infinitely often in every other tiling). The talk is aimed at a general audience.

Date: Monday, June 12, 2017
Time: 15:00
Place: Erdal İNÖNÜ Seminar Hall, Ataköy Campus, İstanbul Kültür University