Oscillation of Delay and Difference Equations with Several Deviating Arguments

Ioannis P. Stavroulakis

Department of Mathematics, University of Ioannina, 451 10 Ioannina, Greece

E-mail: ipstav@uoi.gr

Visiting Professor at Ankara University

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Abstract

Consider the first-order delay differential equation

$$x'(t) + \sum_{i=1}^{m} p_i(t)x(\tau_i(t)) = 0; \qquad t \geq 0,$$

where, for every $i \in \{1, 2, \ldots, m\}$, p_i is a continuous real-valued function in the interval $[0, \infty)$, and τ_i is a real-valued function on $[0, \infty)$ such that

$$\tau_i(t) \leq t \quad \text{for all} \quad t \geq 0 \quad \text{and} \quad \lim_{t\to\infty} \tau_i(t) = \infty,$$

and the discrete analogue difference equation

$$\Delta x(n) + \sum_{i=1}^{m} p_i(n)x(\tau_i(n)) = 0; \qquad n = 0, 1, \ldots,$$

where, for every $i \in \{1, 2, \ldots, m\}$, $\{p_i(n)\}$ are real sequences, and $\{\tau_i(n)\}$ are sequences of integers such that

$$\tau_i(n) \leq n \quad \text{for all} \quad n = 0, 1, \ldots \quad \text{and} \quad \lim_{n\to\infty} \tau_i(n) = \infty.$$

Several optimal oscillation conditions for the above equations are presented.

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