

A Family of Higher Order Iterative Methods for Solving Nonlinear Equations

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Abstract

In this talk, we consider *iterative methods* for the numerical solution of nonlinear equations of the form $f(x) = 0$, where $f : D \subset \mathbb{R} \rightarrow \mathbb{R}$ is a scalar function and D is open. Let $\alpha \in \mathbb{R}$ be a *simple root* of $f(x) = 0$, that is $f(\alpha) = 0$ and $f'(\alpha) \neq 0$, and let $x_0 \in \mathbb{R}$ be an *initial approximation* to the root α . Iterative methods start with x_0 and aim to obtain a sequence $\{x_n\}_{n=0}^{\infty}$ that *converges* to the simple root α . The most widely used one is *Newton's method* defined by $x_{n+1} = g(x_n)$, $n = 0, 1, \dots$, where $g(x) = x - (f(x)/f'(x))$. Its *order of convergence* is 2 for simple roots. In literature some higher order iterative methods based on Newton's or Newton-type methods have been developed to increase the *efficiencies* of the methods. In developing new iterative methods, the aim is, in general, to increase the order of convergence, to minimize the new function evaluations per iteration and hence to increase the efficiencies of the methods, that is, to obtain *optimal methods* in the sense of *Kung-Traub conjecture*. The (family of) methods we shall introduce are based on Newton's method and they are *higher order optimal multipoint methods without memory*. After giving convergence analysis of the methods, numerical examples are given to check the performances and efficiencies and, to verify the theoretical results.

Key Words: Nonlinear equations, Newton's method, iterative methods, order of convergence, efficiency index, informational efficiency.

References

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