Fedja's proof of Deepti's inequality

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The following question has been posted on the Mathoverflow website:
Is it true that, for all $q \in (0,1)$, $k \in \mathbb{N}_0$ and $x \in [0,1]$, the inequality

$$\frac{x^k}{(1-q)(1-q^3)\cdots(1-q^{2k-1})} \leqslant \prod_{j=1}^{\infty} \frac{1}{1-q^{2j-1}x} \tag{1}$$

holds?

The question was posted by a Mathoverflow user, with nickname 'Deepti', and for this reason we call (1) *Deepti's Inequality.* A guideline of the solution to this problem was proposed by the user with username 'fedja'. (See, [1])

In this talk, Deepti's inequality will be proved using fedja's approach. The long way to the elegant proof is divided into several steps each of which is interesting in its own part containing the beauty and aesthetics of mathematics. These steps include problems that can be solved using basic calculus knowledge, quadratures and related error estimates.

References

[1] https://mathoverflow.net/questions/269740/inequality-for-functions-on-0-1

[2] S. Ostrovska, M. Turan, *Fedja's proof of Deepti's inequality*, Turk. J. Math., **42**, (2018), 1091–1097.

[3] J. Stoer, R. Bulirsch, *Introduction to numerical analysis*, Springer-Verlag, New York, 1980.