

Dokuz Eylül University Department of Mathematics



## GENERAL SEMINAR

## On the Tractability of UN/SATISFIABILITY

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**Time:** 13.00 - 14.30

Place: SEMINAR ROOM B206, DEPARTMENT OF MATHEMATICS, DOKUZ EYLÜL UNIVERSITY

## Absract

This paper attacks the **P** vs **NP** problem by means of one-in-three SAT, also known as exactly-1 3SAT (X3SAT). Clauses  $C_k$ , an exactly-1 disjunction  $\odot$  of literals, comprise any X3SAT formula,  $\phi = \bigwedge C_k$ . The assignment  $\phi(r_j) := r_j \land \phi$  denotes that  $r_j$  is true, and reduces by means of  $\odot$  any  $C_k = (r_j \odot \bar{r}_i \odot r_u)$  in  $\phi(r_j)$  into  $\psi_k = (r_j \land r_i \land \bar{r}_u)$  in  $\psi(r_j)$ . Thus,  $\phi(r_j) := r_j \land \phi$  is transformed into  $\phi(r_j) = \psi(r_j) \land \phi'(r_j)$  such that  $\psi(r_j) = \bigwedge \psi_k$  and  $\phi'(r_j) = \bigwedge C_{k'}$ , and that  $\psi(r_j)$  and  $\phi'(r_j)$ are disjoint. If  $\nvDash \psi(r_j)$ , then  $\nvDash \phi(r_j)$ , and  $r_j$  is removed from  $\phi$ . Note that it is trivial to verify if  $\nvDash \psi(r_j)$ , since  $\nvDash \psi(r_j)$  iff  $\psi(r_j)$  includes some  $r_i \land \bar{r}_i$ , and that  $\nvDash \psi(r_j)$ , sufficient for  $\nvDash \phi(r_j)$ , is necessary also, specified in the sequel.  $\psi(r_i)$  is true for all  $r_i$  when every  $r_j$  is removed if  $\nvDash \psi(r_j)$ . Also,  $\psi(r_i) \models \psi(r_i|r_j)$ . Hence,  $\psi(r_i|r_j)$  is true for all  $r_i$  in  $\phi'(r_j)$ , and  $\phi'(r_j)$  is satisfiable. Thus, any X3SAT formula  $\phi(r_i)$  reduces to a conjunction of literals  $\psi(r_i)$ . Therefore, it is tractable via  $\psi(r_i)$  to verify unsatisfiability of  $\phi(r_i)$ , hence to verify satisfiability of  $\phi$ . A satisfiable assignment is then constructed;  $\psi(r_{i_0}) \land \psi(r_{i_1}|r_{i_0}) \land \cdots \land \psi(r_{i_n}|r_{i_m}) = \phi$ . It takes  $O(n^3)$ . Therefore, **P** = **NP**.