Boğaziçi

MATH COLLOQUIUM

The Reed & Zenor Theorem

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The Reed & Zenor Theorem

Abstract: The following question, asked in 1920s, is of fundamental importance:

Question: Which normal Hausdorff spaces are metrizable?

The first answer related to a variant of this question is the following outstanding result of F. B. Jones proved in 1937:

Jones' Theorem: Under CH ($\mathbf{c} = \aleph_1$), every normal and separable Moore (regular and developable) space X is metrizable **!**

Here, a sequence $\{\mathcal{G}_n\}_{n=1}^{\infty}$ of open coverings of X is called a ***development*** for X iff

$$\mathcal{B}_x = \{\mathrm{st}(\mathcal{G}_n, x) : n \in \mathbb{N}\}$$

is a local base for every $x \in X$, where the ***star set*** $\mathrm{st}(\mathcal{A}, x)$ of x according to the family $\mathcal{A}$ is nothing but

$$\mathrm{st}(\mathcal{A}, x) = \bigcup \{A \in \mathcal{A} : x \in A\}.$$

Regular developable spaces are called **Moore** spaces after Robert Lee Moore, and thereby Jones asked (or, conjectured) the following:

Jones' Conjecture(1937) Every normal Moore space is metrizable **!**

What Jones actually observed was that this was a set-theoretic problem, i.e. it depends on the axioms of a certain model of Set Theory. Further improvements in the topic yielded some interesting results:

Bing's Theorem(1951): A Moore space is metrizable iff it is collectionwise normal.

Fleissner's Theorem (1976): It is consistent that each normal, locally compact Moore space is metrizable **!**

Fitzpatrik & Traylor's Theorem(1966): If there exists a normal separable non-metrizable Moore space, then there exists a locally compact, normal and separable Moore space **!**

Finally, the following serious problem was asked within ZFC:

Question (**Traylor**(1968)): Is every normal locally compact, and locally connected Moore space metrizable?

Traylor's Problem has been solved affirmatively within ZFC by G. M. Reed and P. L. Zenor in 1976.

Providing some important steps in the proof of Reed and Zenor will constitute the subject matter of this talk.

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