THE NUMBER OF SINGULAR FIBERS IN HYPERELLIPTIC LEFSCHETZ FIBRATIONS

Abstract

For the past two decades the category of 4-dimensional has experienced explosive growth. A number of surprising results, which exhibits rich and complicated relationships between different categories of manifold which are unique to dimension 4, has been discovered associated with this growth. These developments have made symplectic 4-manifolds that are another category of 4-dimensional manifolds natural candidates to be the building blocks of all smooth 4-manifolds having infinitely differentiable local identification maps.

Donaldson and Gompf results ([1], [2], [3] and [4]) give the relation between symplectic 4-manifolds and Lefschetz fibrations, which are a fibering of a 4-manifold by surfaces, with a finite number of singularities of a prescribed type. Their results say that symplectic 4-manifolds (after perhaps blowing up) admit the structure of a Lefschetz fibration and a genus-g Lefschetz fibration with a fiber genus $g \geq 2$ over the Riemann surface admits a symplectic structure. Hence, Lefschetz fibrations provide a combinatorial way to study symplectic 4-manifolds. The results on the minimal number of vanishing cycles of a Lefschetz fibration provide some results on symplectic 4- manifolds, which also gives a connection between symplectic topology and geometric group theory.

The information about the number of singular fibers in a Lefschetz fibration provides us important information about the topological invariants of its total space such as $\sigma(X)$, $e(X)$, $c_1^2(X)$ and so on. In addition, it has been known that the number of singular fibers in a Lefschetz fibration can not be arbitrary. So it makes sense to ask what the minimal number of singular fibers in a nontrivial relatively minimal genus-g Lefschetz fibration over the oriented surface of genus h.

In this talk, after giving some results about the minimal number of singular fibers in a nontrivial relatively minimal genus-g Lefschetz fibration over the surface of genus h, we will state our results about it for hyperelliptic Lefschetz fibrations which are a special kind of Lefschetz fibrations.

References

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