ALGEBRA AND NUMBER THEORY
SYMPOSIUM

Abstracts

Contents

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Welcome to Algebra and Number Theory Symposium in Honour of Prof. Dr. Mehpare Bilhan. The symposium is dedicated to Prof. Bilhan because of her retirement in October 2010. We wish her a happy retirement and a fruitful life in the future. She has given to almost all of us some algebra course during our studies at Middle East Technical University. Prof. Bilhan is not only our teacher but also a close friend and sometimes our mother. She is interested in our problems not only in mathematics but also in daily life problems.

We are grateful to all the participants and speakers of this symposium in particular to the ones who come from long distances, Iran and Russia. We understand and appreciate their support to the symposium.

We would like to thank Middle East Technical University Mathematics Department and Scientific and Research Council of Turkey (TÜBİTAK) for all the support they have given. We hope you have a good time during the symposium.

Organizing Committee
13 October 2010

Mehpare Gökalp [Bilhan] was born in 1943 in Vize, Kırklareli, in Turkish Thrace. For high school, she went to Istanbul, where she graduated from Kandilli Kız Lisesi. She then went to Paris on a grant from the Turkish state to study mathematics. She earned her B.Sc. and Ph.D. degrees from Université Paris VI, Pierre-et-Marie-Curie. During those years, Paris was a very lively place to do mathematics. Mehpare Gökalp [Bilhan] took courses from many famous mathematicians, such as Claude Chevalley, Henri Cartan and Pierre Samuel; she also attended some seminars of Jean-Pierre Serre. She wrote her Ph.D. thesis, which was an analytic study of the L-series associated with Hecke characters, under the direction of Roger Descombes. She also met her future husband Saffet Bilhan, who was doing Ph.D. in the Law School in Paris. While they were still students in Paris, they married and had two boys, Haydar and Ömer.

Mehpare Bilhan returned to Turkey in 1974 and started working in Hacettepe University. In those years, at Middle East Technical University, the algebra group was very active, thanks to people like Cahit Arf, Gündüz Ikeda, Cemal Koç and others. She quickly joined the group, and they had very fruitful seminars and established long-term friendships. She prepared her habilitation on density theorems in global fields in 1978. In 1980, with the encouragement of Cahit Arf and Gündüz Ikeda, Mehpare Bilhan came to Middle East Technical University, where she has been based ever since. In those years, she improved her habilitation work on global fields and obtained new results. In the following years, she also worked on class field theory, class number problems, arithmetic progressions of polynomials over finite fields, and Tchebotarev sets.

Professor Bilhan made academic visits to various institutions, in Italy, France, and Egypt. Between 1997 and 2003, she worked for the Feza Gürsey Institute in Istanbul as a part-time researcher. She has published a number of research articles in algebraic number theory and related fields, in national and international journals. She has a textbook in abstract algebra, written with two co-authors. She has given many talks in conferences abroad and in Turkey, and taught courses at international summer schools, graduate summer schools of TÜBİTAK, and also school teachers' training programs.

Among generations of METU Mathematics Department students, Mehpare Bilhan is well known for her wonderful teaching. She has taught a variety of undergraduate and graduate courses at METU. Almost every METU Math Department alumnus took a number theory or algebra course from her; many took several courses. She has supervised four Ph.D. and nine M.Sc. theses. Currently she is directing one Ph.D. thesis.

She is the grandmother of four: Defne, Demir, Kerim, and Zeynep.

Program

October 13, 2010 (Wednesday)

09:00-09:50	Henning Stichtenoth	*On the number of rational points on algebraic curves over finite fields*
10:00-10:50	Cem Güneri	*Rational Points on Curves over Finite Fields*
Coffee Break		
11:20-11:50	İlhan İkeda	*On Eisenstein series*
11:50-12:20	Mahmut Kuzucuoğlu	*Universal Groups*
LUNCH		
14:00-14:50	Ersan Akyıldız	*Generalized Kostant-Macdonald identity and a smoothness condition for Schubert varieties in G/B*
15:00-15:50	Ayşe Berkman	*Group Actions in the Finite Morley Rank Context*
Coffee Break		
16:20-16:50	Ekin Özman	*Points on Quadratic Twists of the Classical Modular Curve*
16:50-17:20	Kıvanç Ersoy	*Locally Finite Groups with Anticentral Elements*
17:20-17:40	Hakan Özadam	*The Hamming distance of cyclic codes of length p^s over $GR(p^2, m)$*
17:40-18:00	Ayberk Zeytin	*Hermitian Lattices and Algebraic Curves*
Cocktail		

October 14, 2010 (Thursday)

09:00-09:50	Mohammad Shahryari	*Character Degrees of Polyadic Groups*
10:00-10:50	Emrah Çakcak	*Fourier coefficients of power permutations in characteristic two and relative cohomology of certain families of curves*
Coffee Break		
11:20-11:50	Hatice Kandamar	*Gamma Rings*
11:50-12:20	Feride Kuzucuoğlu	*On Strongly Prime Right Ideals*
LUNCH		
14:00-14:50	Cem Yalçın Yıldırım	*Small gaps between primes and almost primes*
15:00-15:50	Ali Ulaş Özgür Kişisel	*Products generalizing the factorial*
Coffee Break		
16:00-17:00	Anılar	*Mehpare hocamız ile ilgili anılarınızı paylaşırsanız* *seviniriz.*
Conference Dinner at Eymir Lake		

October 15, 2010 (Friday)

09:00-09:50	Vladimir M. Levchuk	*The normal structure and extremal subgroups in the unipotent subgroup of the Lie type groups*
10:00-10:50	Alev Topuzoğlu	*On permutations of finite fields*
Coffee Break		
11:20-11:50	Sefa Feza Arslan	*New families supporting Rossi's conjecture*
11:50-12:20	Kamal Aziziheris	*Bounding the derived length for a given set of* *character degrees*
LUNCH		
14:00-14:50	Sinan Sertöz	*Counting Lines on Algebraic Surfaces*
15:00-15:50	Ferruh Özbudak	*Quadratic forms in even characteristic and maximal/minimal curves over finite fields*
Coffee Break		
16:20-16:50	Erol Serbest	*Ramification Theory in Non-Abelian Local Class* *Field Theory*
16:50-17:10	Cem Tezer	*Characterisation of* $\mathbb{R}/\nvDash\pi\mathbb{Z}$ *by means of a linear order*
17:20-17:40	Sevgi Harman	*Radically Perfect Ideals in Commutative Rings*
17:40-18:00	Tamer Koşan	*On Modules over Group Rings*
18:00-18:20	Ömer Küçüksakallı	*Class number one problem*

Generalized Kostant-Macdonald identity and a smoothness condition for Schubert varieties in G/B

Ersan AKYILDIZ - James B. CARRELL

By a $\mathfrak{B}$-regular variety, we mean a smooth projective variety over $\mathbb{C}$ admitting an algebraic action of the upper triangular Borel subgroup $\mathfrak{B} \subset SL_2(\mathbb{C})$ such that the unipotent radical in $\mathfrak{B}$ has a unique fixed point. The purpose of this note is to give a refinement of the generalized Kostant-Macdonald identity proved in [$Proc.Nat.Acad.Sci.USA, 1989$] for the Poincaré polynomial of a $\mathfrak{B}$-regular variety. When interpreted for a smooth Schubert variety X in the flag variety G/B of an algebraic group G over $\mathbb{C}$, this yields an amusing identity for the size of the Bruhat interval in the Weyl group of G corresponding to X, which gives a new elementary necessary condition for a Schubert variety in G/B to be smooth.

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New families supporting Rossi's conjecture

Sefa Feza ARSLAN

Rossi's conjecture saying that every Gorenstein local ring has non-decreasing Hilbert function is not solved even for monomial curves. In [1], we show that the Hilbert function is non-decreasing for local Gorenstein rings with embedding dimension four associated to monomial curves, under some arithmetic assumptions on the generators of their defining ideals in the non-complete intersection case. In [2], by using the technique of nice gluing, we give infinitely many families of 1-dimensional local rings associated to complete intersection monomial curves with non-decreasing Hilbert functions. In this talk, by using not nice gluing, we give families of 1-dimensional local rings associated to complete intersection monomial curves given with free parameters supporting Rossi's conjecture [3]. This is a joint work with Nil Şahin and Neslihan Ös Sipahi.

References

[1] F. Arslan, P. Mete, Hilbert functions of Gorenstein monomial curves, Proc. Amer. Math. Soc. 135 (2007) 1993-2002.

[2] F. Arslan, P. Mete, M. Şahin, Gluing and Hilbert functions of monomial curves, Proc. Amer. Math. Soc. 137 (2009) 2225-2232.

[3] F. Arslan, N. Şahin, N. Ös Sipahi, Monomial curves obtained by not nice gluing, Preprint

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Bounding the derived length for a given set of character degrees

Kamal AZIZIHERIS

Let G be a finite solvable group with $\{1, a, b, c, ab, ac\}$ as the character degree set, where a, b, and c are pairwise coprime integers greater than 1. We show that the derived length of G is at most 4. This verifies that the Taketa inequality, $\mathrm{dl}(G) \leq |\mathrm{cd}(G)|$, is valid for solvable groups with $\{1, a, b, c, ab, ac\}$ as the character degree set. Also, as a corollary, we conclude that if a, b, c, and d are pairwise coprime integers greater than 1 and G is a solvable group such that $\mathrm{cd}(G) = \{1, a, b, c, d, ac, ad, bc, bd\}$, then $\mathrm{dl}(G) \leq 5$. Finally, we construct a family of solvable groups whose derived lengths are 4 and character degree sets are in the form $\{1, p, b, pb, q^p, pq^p\}$, where p is a prime, q is a prime power of an odd prime, and $b > 1$ is integer such that p, q, and b are pairwise coprime. Hence, the bound 4 is the best bound for the derived length of solvable groups whose character degree set is in the form $\{1, a, b, c, ab, ac\}$ for some pairwise coprime integers a, b, and c.

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Group Actions in the Finite Morley Rank Context

Ayşe BERKMAN

All groups are assumed to be infinite, and of finite Morley rank, and all actions are assumed to be definable. In my talk, I shall focus on groups acting on abelian groups. The following classical result in this setting was proven by Boris Zilber.

Theorem. (Zilber) Let G be connected group with an infinite center, acting faithfully and minimally on a connected abelian group V. Then V is a vector space over an algebraically closed field, and G is a subgroup of $\mathrm{GL}(V)$ in its natural action on V.

Alexandre Borovik and Gregory Cherlin asked the following.

Question. (Borovik – Cherlin) Let G be a connected group acting irreducibly, faithfully, and with pseudoreflection subgroups on a connected abelian group V. Then is V a vector space, and is G isomorphic to $GL(V)$ in its natural action?

In my talk, I shall present a possible approach to this question, and give an affirmative answer for the case where the pseudorank of G is equal to the Morley rank of V.

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Fourier coefficients of power permutations in characteristic two and relative cohomology of certain families of curves

Emrah ÇAKÇAK

This is an ongoing joint work with Philippe Langevin. We consider a certain family of Artin-Schreier curves related to Fourier coefficients of power permutations on a finite field of characteristic two. Here, using p-adic cohomology and deformation techniques, intoduced by several authors, we determine the Picard-Fuchs equation satisfied by the Frobenius endomorphism acting on the relative Monsky-Washnitzer cohomology of these curves.

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Rational Points on Curves over Finite Fields

Cem GÜNERİ

Let X be an algebraic curve of genus g defined over the finite field $\mathbb{F}_q$ with q elements. The number $N_q(X)$ of $\mathbb{F}_q$-rational points on X is bounded by the celebrated Hasse-Weil theorem:

$$N_q(X) \leq q + 1 + 2g\sqrt{q}.$$

We plan to talk about the number of rational points of curves over finite fields and, if time permits, their applications in coding theory. Special emphasis will be on curves reaching the Hasse-Weil bound, the so-called maximal curves.

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On Eisenstein series

Kazım İlhan İKEDA

To my teacher Mehpare Bilhan

Introduction In the theory of elliptic modular forms, there are two ways to construct Eisenstein series. The first way is to define Eisenstein series $E_L(z,s;N)$ via a "lattice" L, where for simplicity we assume for the time being that $L=\mathbb{Z}\times\mathbb{Z}$, and a certain "congruence relation" imposed on the lattice L defined modulo N for some $0<N\in\mathbb{Z}$, where again for simplicity we let $N=1$, as follows. For $z\in\mathbb{H}_1=\{z\in\mathbb{C}:\mathrm{im}(z)>0\}$ the Poincaré upper-half plane and for $s\in\mathbb{C}$ with $\mathrm{Re}(s)>1$, define

$$E_L(z,s)=\frac{1}{2}\sum_{(0,0)\neq(m,n)\in L}\delta(z)^s J_{2s}[(m,n),z]^{-1}, \tag{$*$}$$

where $J_{2s}[(m,n),z]:=\mid mz+n\mid^{2s}$ for $(0,0)\neq(m,n)\in L$, $z\in\mathbb{H}_1$, and $\delta(z)=\mathrm{im}(z)$ for $z\in\mathbb{H}_1$. The second way to define Eisenstein series $E_P(z,s;\Gamma)$ is with respect to a "parabolic subgroup" P of $G=SL_2(\mathbb{Q})$, where for simplicity, we assume that $P=\left\{\begin{pmatrix} * & * \\ 0 & * \end{pmatrix}\in G\right\}$, and a certain "congruence subgroup" Γ of level N in $SL_2(\mathbb{Z})$ for some $0<N\in\mathbb{Z}$, where again for simplicity, we choose $N=1$ which means Γ to be the full modular group $SL_2(\mathbb{Z})$, as follows. For $z\in\mathbb{H}_1$ and for $s\in\mathbb{C}$ with $\mathrm{Re}(s)>1$, define the sum by

$$E_P(z,s)=\sum_{\alpha\in(\Gamma\cap P)\backslash\Gamma}\mathrm{im}(\alpha.z)^s, \tag{$\dagger$}$$

where the action $(\alpha,z)\mapsto\alpha.z$ of $\alpha=\begin{pmatrix} a_\alpha & b_\alpha \\ c_\alpha & d_\alpha \end{pmatrix}\in\Gamma$ on $z\in\mathbb{H}_1$ is defined as usual by $\alpha.z=\frac{a_\alpha z+b_\alpha}{c_\alpha z+d_\alpha}\in\mathbb{H}_1$. The imaginary part $\mathrm{im}(\alpha.z)$ of $\alpha.z\in\mathbb{H}_1$, for $\alpha\in\Gamma$ and $z\in\mathbb{H}_1$, is $\mathrm{im}(\alpha.z)=\frac{\delta(z)}{|c_\alpha z+d_\alpha|^2}$. The Eisenstein series defined by ($\dagger$) is well-defined. In fact, let $\alpha\in\Gamma$ and $p\in P\cap\Gamma$, let $z\in\mathbb{H}_1$. Now,

$$(p\alpha).z=p(\alpha.z)=\pm(\pm\alpha.z+b_p)=\alpha.z\pm b_p.$$

Thus, $\mathrm{im}((p\alpha).z)=\mathrm{im}(\alpha.z)$. We can rewrite the Eisenstein series ($\dagger$), for $z\in\mathbb{H}_1$ and $s\in\mathbb{C}$ with $\mathrm{Re}(s)>1$, as

$$E_P(z,s)=\sum_{\alpha\in\Gamma_\infty\backslash\Gamma}\delta(z)^s J_{2s}(\alpha,z)^{-1}, \tag{$\ddagger$}$$

where $J_{2s}:G\times\mathbb{H}_1\to\mathbb{C}$ is a $\mathbb{C}$-valued factor of automorphy defined by $J_{2s}(\alpha,z)=\mid c_\alpha z+d_\alpha\mid^{2s}$ for $\alpha\in G$ and $z\in\mathbb{H}_1$. Here, Γ_∞ is defined as usual by $\Gamma_\infty=P\cap\Gamma$. Note that, by the cocycle relation satisfied by the factor of automorphy $J_{2s}:G\times\mathbb{H}_1\to\mathbb{C}$, for $p\in\Gamma_\infty$, $\alpha\in\Gamma$ and $z\in\mathbb{H}_1$, $J_{2s}(p\alpha,z)=J_{2s}(p,\alpha.z)J_{2s}(\alpha,z)=J_{2s}(\alpha,z)$. Thus, $J_{2s}:G\times\mathbb{H}_1\to\mathbb{C}$ induces a well-defined $\mathbb{C}$-valued function on $\Gamma_\infty\backslash\Gamma\times\mathbb{H}_1$. Both of the series ($*$) and ($\dagger$) or ($\ddagger$) are well-defined and converge absolutely and uniformly defining functions which are holomorphic in s and real-analytic and Γ-invariant with respect to z.

Among these two types of Eisenstein series, the first type, namely the function $E_L(z,s)$ has *better analytic properties* than the second type function $E_P(z,s)$. In fact, following [Zag], we state the following theorem without giving a proof.

Theorem 1. *The Eisenstein series $E_L(z,s)$ can be continued meromorphically to the whole complex s-plane, which is holomorphic except for simple poles at $s=0$ and $s=1$ with residues $-\frac{1}{2}$ and $\frac{1}{2}$ respectively, and satisfies the following functional equation*

$$E_L^*(z,s) = E_L^*(z,1-s), \tag{§}$$

where

$$E_L^*(z,s) = \pi^{-s}\Gamma(s)E_L(z,s), \tag{¶}$$

which is called the completed Eisenstein series.

On the other hand, for *Langlands functoriality*, in particular to study Langlands L-functions via Rankin-Selberg or Langlands-Shahidi methods, the second type Eisenstein series $E_P(z,s)$ is much more suitable than $E_L(z,s)$. For example, again following [Zag], the Rankin zeta-function $R_{f_j}(s)$ defined for a Maass eigenform $f_j(z)$ by

$$R_{f_j}(s) = \frac{\Gamma(s/2)^2}{8\pi^s\Gamma(s)}\Gamma\left(\frac{s}{2}+ir_j\right)\Gamma\left(\frac{s}{2}-ir_j\right)\sum_{\substack{n=-\infty\\ n\neq 0}}^{\infty}\frac{\mid a_j(n)\mid^2}{\mid n\mid^s}$$

for $s\in\mathbb{C}$ with $\mathrm{Re}(s)>1$, can be obtained by integrating the specific function $\mid f_j(z)\mid^2$, where $f_j(z)$ is the Maass eigenform, against the Eisenstein series $E_P(z,s)$. Recall that, for any complex-valued continuous Γ-invariant function $f:\mathbb{H}_1\to\mathbb{C}$ on the upper-half plane $\mathbb{H}_1$ has a Fourier expansion of the form $f(z)=\sum_{n=-\infty}^{\infty}a_n(f;y)e^{2\pi inx}$, where $z=x+iy\in\mathbb{H}_1$ with $x=\mathrm{Re}(z)$ and $y=\mathrm{im}(z)$. As usual, let $L^2(\Gamma\backslash\mathbb{H}_1)$ be the Hilbert space of complex-valued square-integrable Γ-invariant functions, and let $L_0^2(\Gamma\backslash\mathbb{H}_1)$ be the subspace of functions $f:\mathbb{H}_1\to\mathbb{C}$ with $a_n(f;y)=0$. Let $\Delta=y^2\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)$ be the Laplace operator on $L^2(\Gamma\backslash\mathbb{H}_1)$. The subspace $L_0(\Gamma\backslash\mathbb{H}_1)$ is stable under the oparator Δ, and has a basis $\{f_j\}_{1\leq j\in\mathbb{Z}}$ consisting of eigenforms of Δ satisfying $\Delta f_j=-\left(\frac{1}{4}+r_j^2\right)f_j$, where $r_j\in\mathbb{C}$ and $\frac{1}{4}+r_j^2\geq 0$, for each $j=1,2,\cdots$. Thus, the n^{th} Fourier coefficient $a_n(f_j;y)$ satisfies the second-order differential equation

$$y^2\frac{d^2}{dy^2}a_n(f_j;y)-4\pi^2n^2y^2a_n(f_j;y) = -\left(\frac{1}{4}+r_j^2\right)a_n(f_j;y).$$

It follows that $a_n(f_j;y)=\sqrt{y}K_{ir_j}(2\pi\mid n\mid y)$ is the unique solution of this differential equation, which remains bounded as $y\to\infty$, where for $\nu\in\mathbb{C}$, the K-Bessel function $K_\nu(z)$ is defined by

$$K_\nu(z) = \int_0^\infty e^{z\cosh(t)}\cosh(\nu)t\,dt,$$

for $z\in\mathbb{C}$ with $\mathrm{Re}(z)>0$.

In fact, for any complex-valued Γ-invariant function $f:\Gamma\backslash\mathbb{H}_1\to\mathbb{C}$ on the upper-half plane $\mathbb{H}_1$ which is of sufficiently rapid decay, for example $f(z)=O(y^{-\varepsilon})$ as $y\to\infty$ for some $\varepsilon>0$, the scalar product

$$\langle f\mid E_P(.,\overline{s})\rangle := \int_{\Gamma\backslash\mathbb{H}_1} f(z)E_P(z,s)dz$$

converges absolutely for some s satisfying $\mathrm{Re}(s)>1$. For such s, the scalar product $\langle f\mid E_P(.,\overline{s})\rangle$ has an integral representation as

$$\langle f\mid E_P(.,\overline{s})\rangle = \int_0^\infty A_0(f;y)y^{s-2}dy,$$

where the term $A_0(f;y)$ is defined by the Fourier expansion of the function $f:\mathbb{H}_1\to\mathbb{C}$ of the form

$$f(z)=\sum_{n=-\infty}^{\infty}A_n(f;y)e^{2\pi inx},$$

where $z=x+iy\in\mathbb{H}_1$, as $f:\mathbb{H}_1\to\mathbb{C}$ is a continuous and Γ-invariant function. In particular, choosing $f(z)=\mid f_j(z)\mid^2$, where $f_j(z)$ is a Maass eigenform, the constant term of $f(z)$ is

$$A_0(f;y)=y\sum_{n\neq 0}\mid a_j(n)\mid^2 K_{ir_j}(2\pi\mid n\mid y)^2,$$

where $a_j(n)\in\mathbb{C}$, $r_j\in\mathbb{R}$ with $r_j\in\mathbb{R}$, and $K_{ir_j}(2\pi\mid n\mid y)$ is real, where for $\nu\in\mathbb{C}$, the K-Bessel function is defined by

$$K_\nu(z)=\int_0^\infty e^{z\cosh(t)}\cosh(\nu)tdt,$$

for $z\in\mathbb{C}$ with $\mathrm{Re}(z)>0$. Thus, the scalar product $\langle f\mid E_P(.,\overline{s})\rangle$ is

$$\begin{aligned}\langle f\mid E_P(.,\overline{s})\rangle&=\int_{\Gamma\backslash\mathbb{H}_1}\mid f_i(z)\mid^2 E_P(z,s)dz\\&=\int_0^\infty y^{s-1}\sum_{n\neq 0}\mid a_j(n)\mid^2 K_{ir_j}(2\pi\mid n\mid y)^2dy\\&=\sum_{n\neq 0}\frac{\mid a_j(n)\mid^2}{\mid n\mid^s}\int_0^\infty y^{s-1}K_{ir_j}(2\pi y)^2dy\\&=R_{f_j}(s),\end{aligned}$$

which is the Rankin zeta-function.

The Eisenstein series $E_L(z,s)$ and $E_P(z,s)$ are related with each other; that is, *linearly equivalent*, by the following simple equality.

$$\zeta_{\mathbb{Q}}(2s)E_P(z,s)=E_L(z,s).\qquad(\|)$$

In fact, there exists a bijection

$$\Gamma_\infty\backslash\Gamma\to\{\pm(c,d):\gcd(c,d)=1\}$$

defined by

$$\Gamma_\infty\begin{pmatrix}a&b\\c&d\end{pmatrix}\mapsto\pm(c,d)$$

for every $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma$. Thus, for $z \in \mathbb{H}_1$ and $s \in \mathbb{C}$ with $\mathrm{Re}(z) > 1$,

$$\begin{aligned}
\zeta_{\mathbb{Q}}(2s) E_P(z,s) &= \sum_{n=1}^{\infty} \frac{1}{n^{2s}} \sum_{\alpha \in \Gamma_\infty \backslash \Gamma} \delta(z)^s J_{2s}(\alpha, z)^{-1} \\
&= \sum_{n=1}^{\infty} \frac{1}{n^{2s}} \sum_{\alpha \in \Gamma_\infty \backslash \Gamma} \delta(z)^s \mid c_\alpha z + d_\alpha \mid^{-2s} \\
&= \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n^{2s}} \sum_{\substack{(c,d) \in L \\ \gcd(c,d)=1}} \delta(z)^s \mid cz + d \mid^{-2s} \\
&= \frac{1}{2} \sum_{n=1}^{\infty} \sum_{\substack{(c,d) \in L \\ \gcd(c,d)=1}} \delta(z)^s (n \mid cz + d \mid)^{-2s} \\
&= \frac{1}{2} \sum_{(0,0) \neq (m,n) \in L} \delta(z)^s \mid mz + n \mid^{-2s} \\
&= E_L(z,s).
\end{aligned}$$

However, we should note that, in the adèlic setting the *linear equivalence* between the adèlic versions of Eisenstein series $E_L(z,s)$ and $E_P(z,s)$ is not that simple and straightforward as ($\|$) and its proof. In fact, following closely [JacZag], [GelJac] and [JacLan], let $G = GL(2)$, $Z =$ the center of G, $A = \left\{ \begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix} \in G \right\}$, $N = \left\{ \begin{pmatrix} 1 & * \\ 0 & 1 \end{pmatrix} \in G \right\}$, and $P = AN = \left\{ \begin{pmatrix} a & * \\ 0 & d \end{pmatrix} \in G \right\}$. For a number field F, let $F_{\mathbb{A}} = \mathbb{A}$ be the adèle ring of F, and denote G_F, G_ν and $G_{\mathbb{A}}$ for the groups $G(F)$, $G(F_\nu)$ and $G(\mathbb{A})$ respectively, and let $C = \prod_\nu C_\nu$ be the standard maximal compact subgroup of $G_{\mathbb{A}}$. For any two Hecke characters χ_1 and χ_2 of F (which are trivial on the component $\mathbb{R}^\times_{>0}$) and for any $s \in \mathbb{C}$, $G_{\mathbb{A}}$ acts on the space $H(\chi_1, \chi_2, s)$ consisting of all functions $f : G_{\mathbb{A}} \to \mathbb{C}$ satisfying

- $f\left(\begin{pmatrix} a & x \\ 0 & d \end{pmatrix} g \right) = \chi_1(a)\chi_2(d) \mid \frac{a}{d} \mid^s f(g)$, for every $a, d \in \mathbb{A}^\times$, $x \in \mathbb{A}$, and $g \in G_{\mathbb{A}}$;
- $\int_C \mid f(\kappa) \mid^2 d\kappa < \infty$,

by the right translation, thereby defines a representation $\pi_{\chi_1,\chi_2,s}$ of $G_{\mathbb{A}}$ on $H(\chi_1, \chi_2, s)$.

The linear equivalence between $E(g,s;f)$ and $E(g,s;\Phi,\chi_1,\chi_2)$ is then stated by the following theorem, whose proof we refer the reader to Section 1.1 of [JacZag].

Theorem 2 (Jacquet-Zagier). *If $f \in H(\chi_1, \chi_2)$ is K-finite, then*

$$E(g,s;f) = L(2s, \chi_1\chi_2)^{-1} \sum_i P_i(s) E(g,s;\Phi_i,\chi_1,\chi_2),$$

where $\Phi_i \in \mathcal{S}_o(\mathbb{A}^2)$ and each $P_i(s)$ is the reciprocal of a polynomial in s and in q_ν^{-s} for finitely many places ν which has no zeroes in the right-half plane $\mathrm{Re}(s) > 0$.

The aim of this work is to get and study a similar relationship between the Eisenstein series defined over adèlizations of higher-rank groups. In particular, for the following cases.

Let F be a totally-real number field, K a CM field with totally-real subfield F, and B a totally-indefinite quaternion algebra over F. The reductive algebraic groups that will be considered in this paper are of the form

$$G = \begin{cases} \{X \in GL_{2n}(F) : {}^tXJ_nX = J_n\}, \\ \{X \in SL_{2n}(K) : {}^tX^\rho J_nX = J_n\}, \\ \{X \in GL_{2n}(B) : {}^tX^\iota J_nX = J_n\} \end{cases} \tag{$**$}$$

where $\rho \in \mathrm{Gal}(K/F)$ is the non-trivial Galois involution of K over F, $\iota : B \to B$ is the canonical involution of the quaternion algebra B, and the matrix $J_n \in M_{2n}(\mathbb{Q})$ is defined by

$$J_n = \begin{cases} \begin{pmatrix} 0 & -1_n \\ 1_n & 0 \end{pmatrix}, \\ \begin{pmatrix} 0 & 1_n \\ 1_n & 0 \end{pmatrix} \end{cases}$$

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On Class Numbers of Algebraic Function Fields

Hülya İşcan

In the study, given a field F of an algebraic functions of one variable having a finite field K as exact field of constants, the numerator of the zeta function of F is used to find the class number of F. The coefficients $a_5, a_6, a_7, a_8, a_9, a_{10}$ of the numerator of the zeta function of such a field F are calculated. These coefficients are necessary to obtain class number of F.

Secondly, the solution of the class number 3 problem in an algebraic function field of one variable over a finite exact constant field has been studied the all conditions that the class number is 3 have been determined depending on the genus of the function field, number of elements of constant field and the number of prime divisors.

Finally, the class number for a hyperelliptic function field of genus g, constant field $K = GF(p)$ and equation $y^2 = x^n + a(a \in K^*)$ has been obtained as $h = p^g + 1$ where n and p are prime numbers greather than 2 and p is congruent to a primitive root modulo n.

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Gamma Rings

Hatice KANDAMAR

Since Nobusawa introduced the notion of gamma rings in 1964 as a ternary algebraic system, various studies have been done mainly on the structure of gamma rings; for instance, generalizations of Wedderburn Artin theorems, of the Chavalley-jacobson density theorem, and of the radical theory of rings. In this presentation, we will give almost all major results on gamma rings obtained so far. And we will also give a construction of quotient rings of prime gamma rings and some properties of that rings.

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Products generalizing the factorial

Ali Ulaş Özgür KİŞİSEL

Let $p(x)$ be a polynomial with integer coefficients. Let $\Omega(n) = p(1)p(2)\dots p(n)$. We investigate the question whether or not $\Omega(n)$ can be a square, or squarefull, especially when $p(x)$ is a quadratic polynomial.

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On Strongly Prime Right Ideals

Feride KUZUCUOĞLU

Let R be an associative ring and $I(\neq R)$ a right ideal of R. The right ideal I is defined to be strongly prime if for each x and y in R, $xIy \subseteq I$ and $xy \in I$ imply that either $x \in I$ or $y \in I$.

The goal of this work is to prove that the intersection $rad_r(R)$ of all strongly prime right ideals coincides with the largest locally nilpotent ideal of the ring R. Also, we give some characterization of rings through strongly prime right ideal.

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Universal Groups

Mahmut KUZUCUOĞLU

In the history of algebra, universal objects were always in the center of the research area. In this talk we will discuss the universal locally finite groups and regular limit groups. A group is called a **locally finite group** if every finitely generated subgroup is a finite group.

A locally finite group U is called a **universal locally finite group** if:

(a) every finite group can be embedded into U,

(b) any two isomorphic finite subgroups of U are conjugate in U.

We will remind the basic properties of the universal locally finite groups discussed in [2] and the regular limit groups constructed by O. H. Kegel in [3].

It is well known by Cayley's Theorem that, every group can be embedded into a symmetric group by its regular permutation representation. One of the interesting properties of regular representation is that, if we embed a finite group G by its right regular representation into $Sym(G)$, then any two finite isomorphic subgroups of G are conjugate in $Sym(G)$. The first example of a countable universal locally finite group is given by P. Hall in [1], these groups now are called Hall universal groups. Hall universal groups plays the role of a universe for finite groups and countably infinite locally finite groups this explains the name.

Let κ be a given infinite cardinal. There are two questions relating with the universal locally finite groups.

(1) Are there universal locally finite groups of cardinality κ?

(2) Are any two locally finite universal groups of cardinality κ isomorphic?

Hall answered both questions positively for countable universal locally finite groups. For any given uncountable cardinality κ, existence of 2^κ non-isomorphic universal locally finite groups of cardinality κ is given by S. Shelah and A. J. Macintyre in [4]. In particular the answer to the second question is negative.

Let Ω_0 be an infinite set of cardinal $|\Omega_0| = \kappa_0$. Let's denote by $S_1 := Sym(\Omega_0)$ the symmetric group of cardinal $\kappa_1 = 2^{\kappa_0}$ and $\Omega_1 = S_1$. If for $n \in \mathbb{N}$, one has already obtained by induction a set Ω_n of cardinal κ_n, put $S_{n+1} = Sym(\Omega_n)$, $\Omega_{n+1} = S_n$ and embed S_{n+1} into $S_{n+2} := Sym(\Omega_{n+1})$ by the right regular representation. Then $S = \bigcup_{i=1}^{\infty} S_i$ is a regular limit group constructed by O. H. Kegel in [3]. Kegel studied the basic properties of these regular limit groups. In some sense they have similar character as Hall universal group.

Basic Properties of regular limit groups obtained by Kegel

(i) S is a simple group.

(ii) Any two isomorphic finitely generated subgroup of S are conjugate in S.

(iii) Every group G of cardinality less than cardinality of S can be embedded into S. They are universal groups in this sense.

A subgroup B of S is called a **bounded subgroup** if B is contained in S_n for some $n \in \mathbb{N}$.

In infinite group theory, one of the interesting question is that, if G is a given infinite group; whether it has a subgroup isomorphic to itself. One can ask the stronger question: whether Does there exist a subgroup B in G such that $C_G(B)$ is isomorphic to G? We prove that for regular limit groups for any centerless bounded subgroup B in G the answer is positive.

Theorem 1. *(O. H. Kegel, M. Kuzucuoğlu) Let B be a bounded subgroup of a regular limit group S. Then $C_s(B) \cong B \wr S$.*

Moreover regular limit groups satisfy the same property as Hall universal groups namely:

Lemma 2. *(O. H. Kegel, M. Kuzucuoğlu) The regular limit group S can be written as $S = C_m C_m$, product of conjugacy classes C_m of conjugates of elements of order m for any $m \geq 2$.*

Lemma 3. *(O. H. Kegel, M. Kuzucuoğlu) In the regular limit group S every element is a commutator.*

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The Normal Structure and Extremal Subgroups in the Unipotent Subgroup of the Lie type Groups

V.M. LEVCHUK

Taking G to be a Lie type group over a field K and U to be the unipotent radical of a Borel subgroup in G. The normal structure of certain groups U is considered in [1]. The area has been under active investigation since 1970's. Partially this is represented in Kondratiev's survey [2]. In [3], along with the solution problem (1.5) from [2], the description of automorphisms of the group U, which had been know earlier for $charK \neq 2, 3$ (Gibbs, 1970), was completed. The approach of [3] uses the description of maximal abelian normal subgroups of the unitriangular group $UT(n, K)$ and close structural connections of certain groups U and associated Lie rings.

For the purpose of applications to symplectic amalgams and to CFSG revision, C. Parker and P. Rowley [4, 5, 6] studied groups U with an extremal subgroup. We describe all maximal abelian normal subgroups of the group U. This gives a new description of the extremal subgroups in U. In finite groups U, we consider the problem about large abelian subgroups, [2, Problem 1.6].

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Quadratic forms in even characteristic and maximal/minimal curves over finite fields

Ferruh ÖZBUDAK

We present some of our results on quadratic forms of codimension 2 in even characteristic and maximal/minimal curves over finite fields. This is a report on our joint studies with Elif Saygı and Zülfükar Saygı.

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Counting Lines on Algebraic Surfaces

Sinan SERTÖZ

This is going to be mostly an expository talk on certain aspects of counting lines on algebraic surfaces over the complex numbers. The problem originated with Segre's 1943 paper where he produced some proofs using elimination theory and these proofs are not yet superseded by the so called modern techniques. Recently Boissiere and Sarti gave group theoretical methods to counts such lines. The problem of finding the maximal number of lines lying on an algebraic surface is still wide open for degrees beyond four. Employing Grassmannian techniques to the problem may be a decent attempt to match Segre's proofs. Despite its amenability to number theoretical approaches, the realm of K3 surfaces is also void of any satisfactory results in this direction.

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Character Degrees of Polyadic Groups

Mohammad SHAHRYARI

A non-empty set G together with an n-ary operation $f : G^n \to G$ is called an *n-ary groupoid* and is denoted by (G, f).

According to the general convention used in the theory of n-ary systems, the sequence of elements $x_i, x_{i+1}, \ldots, x_j$ is denoted by x_i^j. In the case $j < i$ it is the empty symbol. If $x_{i+1} = x_{i+2} = \ldots = x_{i+t} = x$, then instead of x_{i+1}^{i+t} we write $\overset{(t)}{x}$. In this convention $f(x_1, \ldots, x_n) = f(x_1^n)$ and

$$f(x_1, \ldots, x_i, \underbrace{x, \ldots, x}_{t}, x_{i+t+1}, \ldots, x_n) = f(x_1^i, \overset{(t)}{x}, x_{i+t+1}^n).$$

An n-ary groupoid (G, f) is called (i, j)-*associative*, if

$$f(x_1^{i-1}, f(x_i^{n+i-1}), x_{n+i}^{2n-1}) = f(x_1^{j-1}, f(x_j^{n+j-1}), x_{n+j}^{2n-1}) \tag{††}$$

holds for all $x_1, \ldots, x_{2n-1} \in G$. If this identity holds for all $1 \leqslant i < j \leqslant n$, then we say that the operation f is *associative* and (G, f) is called an *n-ary semigroup*.

If, for all $x_0, x_1, \ldots, x_n \in G$ and fixed $i \in \{1, \ldots, n\}$, there exists an element $z \in G$ such that

$$f(x_1^{i-1}, z, x_{i+1}^n) = x_0, \tag{‡‡}$$

then we say that this equation is *i-solvable* or *solvable at the place i*. If this solution is unique, then we say that (‡‡) is *uniquely i-solvable*.

An n-ary groupoid (G, f) uniquely solvable for all $i = 1, \ldots, n$, is called an *n-ary quasigroup*. An associative n-ary quasigroup is called an *n-ary group* or a *polyadic group*. In the binary case (i.e., for $n = 2$) it is a usual group.

In [1], representation theory of polyadic groups are investigated by the author and W. Dudek. In this talk we show that some of the well-known properties of character degrees in ordinary finite groups, have interesting general forms in the case of polyadic groups. Among these properties, we give two samples here;

Theorem 1. *Let G be a finite n-ary group and χ be an irreducible complex character of G. Then $\chi(1)$ divides $(n-1)|G|$.*

Theorem 2. *Suppose G is finite n-ary group. Then*

$$(n-1)|G| = \sum_{\chi \in Irr(G)} \chi(1)^2.$$

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On the number of rational points on algebraic curves over finite fields

Henning STICHTENOTH

For a curve $\mathcal{C}$ over a finite field $\mathbb{F}_q$ (projective, non-singular, absolutely irreducible) we denote by $g(\mathcal{C})$ (resp. $N(\mathcal{C})$) the genus (resp. the number of $\mathbb{F}_q$-rational points) of $\mathcal{C}$. The classical Hasse-Weil Theorem says that for given q and $g = g(\mathcal{C})$,

$$q + 1 - 2g\sqrt{q} \leq N(\mathcal{C}) \leq q + 1 + 2g\sqrt{q} \ ,$$

i.e. $N(\mathcal{C})$ lies in a finite interval. A lot of effort has been put into improving the upper bound of this interval, partly motivated by applications of curves with 'many' points in coding theory and cryptography, but also since 'the question represents an attractive mathematical challenge' (van der Geer).

In this talk, we change the point of view slightly: we fix the finite field $\mathbb{F}_q$ and a non-negative integer N and ask for all possible values of g such that there exists a curve $\mathcal{C}$ over $\mathbb{F}_q$ of genus g,

having exactly N rational points. As follows immediately from the Hasse-Weil Theorem, g must satisfy the condition

$$g \geq \frac{N - (q+1)}{2\sqrt{q}} \ .$$

Our main result is

Theorem. *Given a finite field $\mathbb{F}_q$ and an integer $N \geq 0$, there is an integer $g_0 \geq 0$ such that for every $g \geq g_0$, there exists a curve $\mathcal{C}$ over $\mathbb{F}_q$ with $g(\mathcal{C}) = g$ and $N(\mathcal{C}) = N$.*

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On some nice permutations of finite fields

Alev TOPUZOĞLU

Three main problems concerning permutations of finite fields are:

* construction of new classes of permutation polynomials * enumeration of special classes * cycle structure of special classes.

We shall address these problems for some ”nice” permutations.

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Small gaps between primes and almost primes

Cem Yalçın YILDIRIM

An overview of the methods and results concerning small gaps between primes will be presented. The counterparts for almost primes and their consequences for the values taken on by certain arithmetical functions will also be noted. The material is from the works with Goldston, Graham and Pintz.

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Locally Finite Groups with Anticentral Elements

Kıvanç ERSOY

Let G be a group. An element a in G satisfying $aG' = a^G$ is called an **anticentral element**. G.

Example. Let $G = UT(3, K)$ be the group of 3×3 upper triangular matrices whose diagonal entries are equal to 1, over an infinite locally finite field K of characteristic p. Consider $a = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \in G$. Now, G' is the subgroup of G consisting of elements $x \in G$ such that $x_{12} = x_{23} = 0$. Then, observe that

$$aG' = \{ \begin{pmatrix} 1 & 1 & y \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} : \quad y \in K\} = a^G.$$

Therefore, a is an anticentral element in G.

In [4], Ladisch proved that every finite group with an anticentral element is solvable. However, there are examples of infinite groups with an anticentral element which are not solvable.

In this work, our first aim is to answer the following question:

Question. *Is every locally finite group with an anticentral element locally solvable?*

First, by using a theorem of Hartley on centralizers in locally finite groups (see [2, Corollary A1]), we observed that if G is a locally finite group with an anticentral element a and if there exists $x \in G$ such that $C_G(x)$ is finite, then G is locally solvable. For groups with an anticentral element of prime order, by using a theorem of Kegel on groups with a splitting automorphism of prime order (see [3]), we obtained the following result:

Proposition 1. *Let G be a group with an anticentral element a of prime order p.*

1. *If $p = 2$ then G' is abelian.*
2. *If $p = 3$ then G' is nilpotent of class at most 3.*
3. *If G is locally finite, then G' is locally nilpotent.*

Moreover, one can obtain the following result as a consequence of a theorem of Philip Hall (see [5, 14.5.3]):

Lemma 2. *Let G be a group with an anticentral element. If $\gamma_n(G)$ is finite for some n (where $\gamma_n(G)$ denotes the n-th term of the lower central series of G), then G is solvable.*

In particular, if G' is finite, then G is solvable.

The following result is a consequence of the basic properties of a group with an anticentral element:

Proposition 3. *Let G be a group with an anticentral element $a \in G$. If $a \in Z_\alpha(G)$ for some ordinal α (where $Z_\alpha(G)$ denotes the α-th term of the transfinitely extended upper central series of G), then G is hypercentral.*

In this work, we considered periodic linear groups and finitary permutation groups containing anticentral elements and we proved the following main results:

Theorem 4. *Let G be a locally finite group with an $\mathbb{F}-$linear commutator subgroup G' where $\mathbb{F}$ is the algebraic closure of $\mathbb{F}_p$. If G has an anticentral element of order $n = mp^s$ where $(m,p) = 1$, then one of the following cases occurs:*

1. *G is solvable.*
2. *$C_{G'}(a)$ has an infinite abelian subgroup of exponent p^k for some $1 \leq k \leq s$.*

In particular, by Theorem 4, every periodic linear group with a semisimple anticentral element is solvable.

Theorem 5. *Let $G \leq Sym(\Omega)$ be a locally finite group with an anticentral element a. If $supp(a)$ is finite, then G is locally solvable.*

Therefore, we deduce that if a group G containing an anticentral element has a finitary permutational representation, then it is solvable. Also we proved the following result about groups with anticentral elements which are both locally and residually finite:

Theorem 6. *Let G be a residually finite and locally finite group with an anticentral element a. Then G is locally solvable.*

A non-perfect group G is called a *Camina group* if every element of $G \backslash G'$ is anticentral. Dark and Scoppola proved in [1] that a finite Camina groups are solvable. There are examples of infinite finitely generated Camina groups constructed by Olshanskii. In this work, we will give a method to construct infinite non-solvable Camina groups with a periodic linear commutator subgroup. Indeed we proved the following:

Theorem 7. *For each connected algebraic group H over an algebraically closed field of characteristic p, there exist countably many non-isomorphic infinite Camina groups G with $G' \cong H$. In particular, if H is semisimple then G is not solvable.*

This is an ongoing study under the supervision of Prof. Mercede Maj and Prof. Patrizia Longobardi from University of Salerno. This study is supported by TÜBİTAK BİDEB 2219 International Post Doctoral Research Fellowship. The speaker thanks TÜBİTAK for the support.

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Radically Perfect Ideals in Commutative Rings

Sevgi HARMAN

We call an ideal I of a commutative ring R radically perfect if among the ideals of R whose radical is equal to the radical of I the one with the least number of generators has this number of generators equal to the height of I ([4], [5], [6]). This is a generalization of the notion of set theoretic complete intersection of ideals in Noetherian rings to rings that need not be Noetherian.

The notion of set theoretic complete intersection was first considered by Kronecker [7] in late 19^{th} century. Since then an enormous amount of research has evolved around these types of questions [1]. Among them still remaining unsolved is the question whether height two ideals in the polynomial ring $K[X, Y, Z]$ are set theoretic complete intersection where K is of characteristic zero. In search of an answer to this conjecture, we consider the following question under which circumstances on an integral domain R can we conclude that all the prime ideals of the polynomial ring $R[X]$ over R are radically perfect? In this talk, I will present conditions on R that give an answer to this question.

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On Modules over Group Rings

Tamer KOŞAN

I will present the main ideas in the joint paper [1] written with Tsiu-Kwen Lee (National Taiwan University, Taiwan) and Yiqiang Zhou (Memorial University of Newfoundland, St. Johns, Canada).

Let M be a right module over a ring R and let G be a group. The set MG of all formal finite sums of the form $\sum_{g\in G} m_g g$ where $m_g \in M$ becomes a right module over the group ring RG under addition and scalar multiplication similar to the addition and multiplication of a group ring. A module M_R is called *semisimple* if M_R is a direct sum of simple R-modules, or equivalently if every submodule of M_R is a direct summand. A ring R is semisimple Artinian iff R is a semisimple right (or left) module over R. The famous Maschke's Theorem states that, for a finite group G, a group ring RG is a semisimple Artinian ring iff R is a semisimple Artinian ring and $|G|$ is invertible in R. Generally, a group ring RG is a semisimple Artinian ring iff R is a semisimple Artinian ring and G is a finite group whose order is invertible in R, and this is called the generalized Maschke Theorem by Connell [2]. The following theorem below is a module-theoretic version of this theorem.

Theorem Let M_R be a nonzero module and let G be a group. The following are equivalent:

(1) MG is a semisimple module over RG.

(2) M_R is a semisimple module and G is a finite group with $|G|^{-1} \in \mathrm{End}_R(M)$.

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Class number one problem

Ömer Küçüksakallı

The class number is an important invariant in algebraic number theory. Its history can be traced back to Fermat, who made a speculation about integers of the form $x^2 + 5y^2$. Prime numbers represented by quadratic forms $x^2 + ny^2$ are closely related with the class number of $Q(\sqrt{-n})$. Many great mathematicians (Euler, Lagrange, Legendre, Gauss, Dirichlet, Dedekind, Hilbert, ...) have made contributions to this classification problem. In this talk we will give a brief history of class number by focusing on related works of these mathematicians

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The Hamming distance of cyclic codes of length p^s over $GR(p^2, m)$

Hakan ÖZADAM

The Hamming distance of all cyclic codes of length 2^s over $\mathbb{Z}_4$ was determined in [2] explicitly. López-Permouth, Özadam, Özbudak and Szabo generalized this to Galois rings of characteristic p^2 via a Groebner basis approach together with some further arguments. More precisely, they explicitly determined the Hamming distance of all cyclic codes of length p^s over any Galois ring of characteristic p^2. In this talk, an overview of this result will be given.

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Points on Quadratic Twists of the Classical Modular Curve

Ekin ÖZMAN

A $\mathbb{Q}$-curve is an elliptic curve which is isogenous to all of its Galois conjugates. It is a mild generalization of an elliptic curve and has many interesting applications such as twisted Fermat type equations. A quadratic $\mathbb{Q}$-curve is a $\mathbb{Q}$-curve for which the smallest field of definition is a quadratic field. Quadratic $\mathbb{Q}$-curves of degree N defined over $K = \mathbb{Q}(\sqrt{d})$ are parametrized by a quadratic twist of the classical modular curve $X_0(N)$. Unlike $X_0(N)$ itself, it is not immediate to say if the twist has any $\mathbb{Q}$-rational points. We will give an answer to the following question which is stated by Ellenberg:

For which K and N does the quadratic twist of $X_0(N)$ have points over every completion of $\mathbb{Q}$?

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Characterisation of $\mathbb{R}/\nvDash\pi\mathbb{Z}$ by means of a linear order

Cem TEZER

There is a natural order on $\mathbb{R}/\nvDash\pi\mathbb{Z}$ which is not compatible with the group structure. Yet it does satisfy a weakened compatibility condition which will be shown to be sufficient to characterise the $\mathbb{R}/\nvDash\pi\mathbb{Z}$ up to an order preserving isomorphism.

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Hermitian Lattices and Algebraic Curves

Ayberk ZEYTİN

Although the idea has it roots in works of Klein, graphs on surfaces had to wait Grothendieck to become popular. These objects are combinatorial in nature, yet they encode arithmetical data, as they admit an action of the absolute Galois group, Gal(Q), which is one of the most central groups of mathematics lying in the intersection of widely studied subjects. In this talk, we will define these objects rigorously, then on a few examples see how the absolute Galois group acts, and finally introduce two Hermitian lattices, that, we claim, may be used to study the mentioned action.

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