Boğaziçi
MATH COLLOQUIUM

Approximation of the Exit Probability of a Stable Markov Modulated Constrained Random Walk

Ali Devin Sezer

METU

Abstract: Let X be the constrained random walk on $\mathbb{Z}_+^2$ having increments $(1,0)$, $(-1,1)$, $(0,-1)$ with jump probabilities $\lambda(M_k)$, $\mu_1(M_k)$, and $\mu_2(M_k)$ where $\{M_k\}$ is an irreducible aperiodic finite state Markov chain. X represents the lengths of two tandem queues with arrival rate $\lambda(M_k)$, and service rates $\mu_1(M_k)$, and $\mu_2(M_k)$. We assume that the average arrival rate with respect to the stationary measure of M is less than the average service rates, i.e., X is assumed stable. Let τ_n be the first time X hits the line $\partial A_n = \{x : x(1)+x(2) = n\}$, i.e., the first time the sum of the components of X equals n. Let Y be the random walk on $\mathbb{Z}\times\mathbb{Z}_+$ (i.e., constrained only on $\partial_2 = \{y \in \mathbb{Z}\times\mathbb{Z}_+ : y(2) = 0\}$) again modulated by M and having increments $(-1,0)$, $(1,1)$, $(0,-1)$ with probabilities $\lambda(M_k)$, $\mu_1(M_k)$, and $\mu_2(M_k)$. Let $B = \{y \in \mathbb{Z}^2 : y(1) = y(2)\}$ and let τ be the first time Y hits B. Let $T_n : \mathbb{Z}^2 \mapsto \mathbb{Z}^2$ be the affine map $y \mapsto (n - y(1), y(2)$ and let m denote the initial point of M. For $x \in \mathbb{R}_+^2$, $x(1)+x(2) < 1$, $x(1) > 0$, and $x_n = \lfloor nx \rfloor$, we show that $P_{(T_n(x_n),m)}(\tau < \infty)$ approximates $P_{(x_n,m)}(\tau_n < \tau_0)$ with exponentially vanishing relative error as $n \to \infty$. For the analysis we define a characteristic matrix in terms of the jump probabilities of (X,M). The 0-level set of the characteristic polynomial of this matrix defines the characteristic surface $\mathcal{H} \subset \mathbb{C}^2$ for the problem. Conjugate points on $\mathcal{H}$ and the associated eigenvectors of the characteristic matrix are used to define (sub/super) harmonic functions which play a fundamental role both in our analysis and the computation / approximation of $P_{(y,m)}(\tau < \infty)$.

*Joint work with Fatma Başoğlu Kabran

Date : Wednesday, March 20, 2019
Time: 13:30
Place: TB 130, Boğaziçi University

We would like to thank IMBM for their hospitality