

The loop space homology of a small category

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In an article published in 2009, Dave Benson described, for a finite group G , the mod p homology of the space $\Omega(BG_p^\wedge)$ — the loop space of the p -completion of BG — in purely algebraic terms. In joint work with Carles Broto and Ran Levi, we have tried to better understand Benson's result by generalizing it. Among other things, we showed that when \mathcal{C} is a small category, $|\mathcal{C}|$ is its geometric realization, R is a commutative ring, and $|\mathcal{C}|_R^+$ is a plus construction of $|\mathcal{C}|$ with respect to homology with coefficients in R , then $H_*(\Omega(|\mathcal{C}|_R^+); R)$ is the homology of any chain complex of projective $R\mathcal{C}$ -modules that satisfies certain conditions. Benson's theorem is then the special case where \mathcal{C} is the category associated to a finite group G and $R = \mathbb{F}_p$, so that p -completion appears as a special case of the plus construction.