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Magnetic geodesics and magnetic maps

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Abstract

Geodesics on a Riemannian manifold (M, g) are given by a second order nonlinear differential equation, obtained as critical point of the *kinetic energy*

$$E(\gamma) = \int \frac{1}{2}|\gamma'(s)|^2 \, ds.$$

In this talk I will show first how the energy functional can be perturbed to obtain other trajectories on a manifold, known as *magnetic curves* or *magnetic geodesics*.

Let now ω be a 1-form called the *potential* 1-*form*. For a smooth curve $\gamma : [a, b] \longrightarrow M$ we consider the functional

$$LH(\gamma) = \int_a^b \left(\frac{1}{2}\langle \gamma'(t), \gamma'(t) \rangle + \omega(\gamma'(t)) \right) dt,$$

often called the *Landau Hall functional* for the curve γ, which is a perturbation of the kinetic energy of the curve with the potential ω. The critical points of the LH functional satisfy the Lorentz equation

$$\nabla_{\gamma'}\gamma' - \phi(\gamma') = 0.$$

Here ϕ is a $(1, 1)$ tensor field on M, called the *Lorentz force* and defined by $g(\phi X, Y) = d\omega(X, Y)$, for all X, Y tangent to M.

The notion of geodesic is generalized to harmonic maps between Riemannian manifolds. The second part of the talk is devoted to the *Landau Hall functional for maps*, as a perturbation of the energy functional of a map:

$$LH(f) = E(f) + \int_N \omega(df(\xi))dv_h.$$

Definition. The map f is called *magnetic* with respect to ξ and ω if it is a critical point of the Landau Hall integral defined above, i.e. the first variation $\frac{d}{d\epsilon}LH(f_\epsilon)\big|_{\epsilon=0}$ is zero. This is equivalent to

$$\tau(f) = \phi(f_*\xi).$$

This notion generalizes both magnetic curves and harmonic maps. It helps us also to define new notions such as magnetic vector fields, magnetic endomorphisms on the tangent bundle, magnetic submanifolds and many other.

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Time: 10:00

Place: IMBM Seminar Room, Boğaziçi University South Campus