Exit Probabilities of Constrained Random Walk

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November 13, 2019

Abstract

Consider a nearest neighbor stable two dimensional random walk X constrained to remain on the positive orthant. X is assumed stable, i.e., its average increment points toward the origin. X represents the lengths of two queues (or two stacks in computer science applications) working in parallel. The probability p_n that the sum of the components of this random walk reaches a high level n before the random walk returns to the origin is a natural performance measure, representing the probability of a buffer overflow in a busy cycle. The stability of the walk implies that p_n decays exponentially in n. Let Y be the same constrained random walk as X, but constrained only on its second component and the jump probabilities on its first component reversed. The present article shows that one can approximate p_n with the probability that components of Y ever equal each other, with exponentially decaying relative error, if X starts from an initial point with nonzero first component. We further construct a class of Y-harmonic functions from single and conjugate points on a characteristic surface, with which the latter probability can be either computed perfectly in some cases, or approximated with bounded relative error in general.

Keywords: approximation of probabilities of rare events, exit probabilities, constrained random walks, queueing systems, large deviations