Mathematics of Turbulent Flows: A Million Dollar Problem!

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Turbulence is a classical physical phenomenon that has been a great challenge to mathematicians, physicists, engineers and computational scientists. In the end of the last century, chaos theory was developed to explore similar phenomena that occur in a wide range of applied sciences, but the eyes have always been on the big ball – Turbulence. Controlling and identifying the onset of turbulence have a great economic and industrial impact ranging from reducing the drag on cars and commercial airplanes to better design of fuel engines, weather and climate predictions.

It is widely accepted by the scientific community that turbulent flows are governed by the Navier-Stokes equations, for large Reynolds numbers, i.e. when the nonlinear advective effects dominate the linear viscous effects (internal friction within the fluids) in the Navier-Stokes equations. As such, the Navier-Stokes equations form the main building block in any fluid model, in particular in global climate models. Whether the solutions to the three-dimensional Navier-Stokes equations remain smooth, indefinitely in time, is one of the most challenging mathematical problems. Therefore, by the turn of the millennium, it was identified by the Clay Institute of Mathematics as one of the seven most outstanding Millennium Problems in mathematics, and it has set one million US dollars prize for solving it. Notably, reliable computer simulations of turbulent flows is way out of reach even for the most powerful state-of-the art supercomputers. In this talk I will describe, using layman language, the main challenges that the different scientific communities are facing while attempting to attack this problem. In particular, I will emphasize the mathematical point of view of turbulence.