



The Dade group of a finite group and dimension functions

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Abstract: Let G be a finite group and k an algebraically closed field of characteristic $p > 0$. We define the notion of a Dade kG -module as a generalization of endopermutation modules for p -groups. We show that under a suitable equivalence relation, the set of equivalence classes of Dade kG -modules forms a group under tensor product, and the group obtained this way is isomorphic to the Dade group $D(G)$ defined by Lassueur [2].

We also consider the subgroup $D^\Omega(G)$ of $D(G)$ generated by relative syzygies ΩX , where X is a finite G -set. Let $C(G; p)$ denote the group of superclass functions defined on the p -subgroups of G . There are natural generators ω_X of $C(G; p)$. We prove that there is a well-defined group homomorphism $\psi_G : C(G; p) \rightarrow D^\Omega(G)$ that sends ω_X to Ω_X .

The main theorem is the verification that the subgroup of $C(G; p)$ consisting of the dimension functions of k -orientable real representations of G lies in the kernel of ψ_G . In the proof we consider Moore G -spaces which are the equivariant versions of spaces which have nonzero reduced homology in only one dimension.

This talk is about a theorem in modular representation theory whose proof is topological using equivariant homotopy theory and homological algebra over orbit category. I will give all necessary definitions to make it possible to follow the talk and provide examples to motivate the theorems.

This is a joint work with Matthew Gelvin [1].

References

- [1] M. Gelvin and E. Yalçın, Dade Groups for Finite Groups and Dimension Functions, preprint, 2020 (arXiv:2007.05322v2).
- [2] C. Lassueur, The Dade group of a finite group, J. Pure Appl. Algebra, 217 (2013), 97-113.

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