

### Mathematics Department 25th Year Seminars

# Dual Baer Criterion and R-projectivity of injective modules

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**Abstract:** Let R be a ring with unity and Mod-R be the category of right R-modules. The Baer's Criterion for injectivity states that a right module M is injective iff it is R-injective, that is for each right ideal I of R, any homomorphism from I into M extends to R. Dually, a right module P is R-projective if for each right ideal I of R any homomorphism from M into R/I lifts to R. Unlike the case for injectivity, R-projective modules need not be projective. That is, the Dual Baer Criterion (DBC, for short) does not hold over every ring. The rings R for which the DBC holds in Mod-R are called right testing. From [4], it is known that right perfect rings are right testing. In [3], Faith stated the characterization of all right testing rings as an open problem. Recently in [6], Trlifaj proved that, the problem of characterizing right testing rings is undecidable in ZFC.

In this talk, after summarizing the aforementioned results, I will mention an extend of the notion of R-projectivity, and discuss some problems related to the rings whose injective right modules are R-projective which are partially solved in [1].

#### References

- [1] Y. Alagöz and E. Büyükaşık, Max-projective modules, J. Algebra Appl. 20 (2021), no. 6. 2150095.
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- [3] C. Faith, Algebra. II, Springer-Verlag, Berlin-New York, 1976. Ring theory, Grundlehren der Mathematischen Wissenschaften, No. 191.
- [4] F. L. Sandomierski, Relative injectivity and projectivity, 1964. Thesis (Ph.D.) The Pennsylvania State University.
- [5] J. Trlifaj, Whitehead test modules, Trans. Amer. Math. Soc. 348 (1996), no. 4, 1521-1554.
- [6] J. Trlifaj, Faith's problem on R-projectivity is undecidable, Proc. Amer. Math. Soc. 147 (2019), no. 2, 497-504.
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