On Decomposition of the Hamiltonian Realization of Vlasov's Plasma

Oğul Esen

This talk is on non-relativistic collisionless plasma particles occupying in a finite region $\mathcal{Q} \subset R^3$. Vlasov's plasma dynamics is determined by the evolution of the plasma density function f, defined on the cotangent bunlde $T^*\mathcal{Q}$ with the Darboux' coordinates $(\mathbf{q}, \mathbf{p})$, according to the Vlasov equation

$$\frac{\partial f}{\partial t} + \frac{1}{m}\mathbf{p} \cdot \nabla_q f - e\nabla_q \phi \cdot \nabla_p f = 0. \tag{1}$$

Here, ϕ is the potential function, m is the mass, and e is the electrical charge. One may couple the Vlasov equation with the Poisson equation

$$\nabla^2 \phi(\mathbf{q}) = -e \int f(q,p) d\mathbf{p}, \tag{2}$$

and arrive at the Vlasov-Poisson equations. Hamiltonian analysis of the Vlasov plasma was achieved in [1]. Kinetic moments of the plasma density functions are determined to be Poisson mappings [2]. The first aim is to provide Hamiltonian (Lie-Poisson) analysis of the Vlasov plasma, and the dynamics of its kinetic moments, from the matched pair decomposition point of view [3]. Both of these physical systems will be recasted as Lie-Poisson systems as couplings of *mutually interacting* (Lie-Poisson) subdynamics [5]. In each case, we observe that one of the constitutive subdynamics is the compressible isentropic fluid flow, and the other is the higher-order (≥ 2) kinetic moments. In this regard, the algebraic/geometric (matched pair) decomposition is in a harmony with the physical intuition. As a second goal, we address another phenomenon in plasma dynamics, namely all possible decompositions of Hamiltonian formulation of BBGKY (Bogoliubov-Born-Green-Kirkwood-Yvon) hierarchy [4] of the order 3. This talk contains some results from [5] and [6].

References

[1] J. E. Marsden and A. Weinstein. The Hamiltonian structure of the Maxwell-Vlasov equations. Phys. D, 4(3):394–406, 1981/82.

[2] J. Gibbons, D. D. Holm, and C. Tronci. Geometry of Vlasov kinetic moments: a bosonic Fock space for the symmetric Schouten bracket. Phys. Lett. A, 372(23):4184–4196, 2008.

[3] O. Esen and S. Sütlü. Hamiltonian dynamics on matched pairs. Int. J. Geom. Methods Mod. Phys., 13(10):1650128, 24, 2016.

[4] J. E. Marsden, P. J. Morrison, and A. Weinstein. The Hamiltonian structure of the BBGKY hierarchy equations. In Fluids and plasmas: geometry and dynamics (Boulder, Colo., 1983), volume 28 of Contemp. Math., pages 115–124. Amer. Math. Soc., Providence, RI, 1984.

[5] O. Esen and S. Sütlü. Matched pair analysis of the Vlasov plasma. arXiv preprint arXiv:2004.12595, 2020.

[6] O. Esen, G. Özcan, and S. Sütlü. On extensions, lie-poisson systems, and dissipations. arXiv preprint arXiv:2101.03951, 2021.