# Border Bases and Border Basis Schemes

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ABSTRACT

The basic idea of border basis theory is to describe a zero-dimensional quotient ring by an order ideal of terms $\mathcal{O}$ whose residue classes form a K-vector space basis of that ring.

In this talk, we compare Gröbner Bases with Border Bases and discuss the advantages of Border Bases. We then introduce Border basis schemes which are schemes that parametrize all zero-dimensional ideals that have an $\mathcal{O}$-border basis. If an order ideal $\mathcal{O}$ with $\mu$ elements is defined in a two dimensional polynomial ring and it is of some special shapes, then the $\mathcal{O}$-border basis scheme is isomorphic to the affine space $\mathbb{A}^{2\mu}$.

We present a general condition for an $\mathcal{O}$-border basis scheme to be isomorphic to an affine space that is independent of the shape of the order ideal and the dimension of the polynomial ring that the order ideal is defined in.

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