

Bahçeşehir University, Istanbul, Turkey
Analysis & PDE Center, Ghent University, Ghent, Belgium
Institute Mathematics & Math. Modeling, Almaty, Kazakhstan

“Analysis and Applied Mathematics”

Weekly Online Seminar

Seminar leaders:

Prof. Allaberen Ashyralyev (BAU, Istanbul),
Prof. Michael Ruzhansky (UGent, Ghent),
Prof. Makhmud Sadybekov (IMMM, Almaty)

Date: **Tuesday, May 10, 2022**

Time: 14.00-15.00 (Istanbul) = 13.00-14.00 (Ghent) = 17.00-18.00 (Almaty)

Zoom link: <https://us02web.zoom.us/j/6678270445?pwd=SFNmQUlVTD0tRaH-IDaVYrN3I5bzJVQT09>, **Conference ID:** 667 827 0445, **Access code:** 1

Speaker:

Prof. Dr. Veli B. Shakhmurov

Antalya Bilim University, Antalya, Turkey

Title: **The Cauchy problems for nonlocal abstract Schrödinger equations and applications**

Abstract: Here, the Cauchy problems for linear and nonlinear nonlocal Schrödinger equations are studied. The equation involves convolution integral operators with general kernel operator functions whose Fourier transforms are operator functions defined in a Banach space E together with some growth conditions. By assuming enough smoothness on the initial data and the operator functions, the local and global existence and uniqueness of solutions are established. We can obtain different classes of nonlocal Schrödinger equations, by choosing the space E and linear operators, which occur in a wide variety of physical systems.

The aim here, is to study the existence, uniqueness and regularity properties of solution of the initial value problem (IVP) for nonlocal nonlinear Schrödinger equation (NSE),

$$i\partial_t u + a\Delta u + A * u = \Delta[B * f(u)], t \in (0, T), x \in \mathbb{R}^n, \quad (1)$$

$$u(x, 0) = \varphi(x) \text{ for a.e. } x \in \mathbb{R}^n, \quad (2)$$

where $A = A(x)$, $B = B(x)$ are linear and nonlinear operator functions in a Hilbert space H , respectively, a is a complex number, $T \in (0, \infty]$, $f(u)$ is a given nonlinear function and $\varphi(x)$ is a given E -valued function. The existence and regularity properties of solutions of IVP for Schrödinger equations (SE) were studied e.g in [1-4] and the references therein. The construction of general solutions of nonlocal SE was studied e.g. in [5-7]. Also, the existence and uniqueness of solutions of IVP for abstract SE were investigated in [8, 9]. Moreover, abstract evolution equations were studied e.g. in [10-17] and the references therein. In contrast to the above mentioned results, we will study the existence, uniqueness of local and global solution of (1)-(2). Moreover, in this paper, the method of proofs naturally differs from those used in previous works. Indeed, since the problem includes an abstract operator in the leading part

and the problem is considered in E -valued L^p -spaces, we need some extra mathematics tools for deriving considered conclusions. For this reason, in the proof we use modern analysis tools such as the following: (1) Operator-valued Fourier multiplier theorems in abstract L^p spaces; (2) Embedding and trace theorems in Banach space valued Sobolev-Lions and Besov-Lions spaces; (3) Theory of semigroups of linear operators in Banach spaces; (4) Perturbation theory of operators; (5) Interpolation of Banach Spaces.

The L^p -regularity properties of (1)-(2) depend crucially on the presence of suitable kernel. Then the question that naturally arises is which of the possible forms of the operator kernel functions are relevant for the global well-posedness of (1)-(2). In this study, as a partial answer to this question, we derive the L^p -well posedness of the corresponding linear problem. By choosing the space E and operators A, B , we obtain different classes of nonlocal SEs which occur in applications.

Biography:

Veli Shakhmurov received his Bachelor degree from Faculty of Mathematics at Azerbaijan State University Tusy, Baku in 1973. He received his PhD degree in the field of Mathematical Analysis from the Academy of Sciences of Azerbaijan in 1979. He received his Doktor of Science degree from V. Steklov Mathematical Institute of Akademy of Sciences USSR, Moscow in 1987. He worked at the Department of Mathematical Analysis of Azerbaijan State University Tusy (1973-1982), Department of High Mathematics of Azerbaijan State Engineering University (1982-1989), Department of Mathematical Analysis of Baky State University (1989-1997), Department of Mathematics of Akdeniz University, Turkey (1993-1994), Engineering Faculty of Istanbul University, Turkey (1997-2007), Engineering Faculty of Istanbul Okan University, Turkey (2007-2020). From 2021 to present time Veli Shakhmurov works at the Antalya Bilim University, Turkey.

Prof. Dr. Veli Shakhmurov has published about 200 scientific papers in different International journals and 5 books in Linear Algebra. He conducts active scientific research in the following areas of mathematics: Abstract harmonic analysis; Operator-valued singular integral operators and multiplier theorems in abstract function spaces; Embedding and trace theorems in Sobolev-Lions and Besov-Lions type spaces of vector-valued functions; Compactness of embedding and estimates of approximation numbers of embedding operators in abstract function spaces; Separability properties of boundary value problems for operator-differential equations in Banach spaces; Well-posedness of abstract parabolic equations; Maximal regularity properties of linear and nonlinear abstract differential equations with small parameters; Maximal regularity properties of linear and nonlinear degenerate operator-differential equations; Maximal regularity properties of linear and nonlinear convolution elliptic equations in Banach spaces; Carleman estimates and unique continuation results for abstract differential equations; Spectral properties of differential operators; Maximal regularity properties of abstract Stokes operators in L^p spaces; Navier-Stokes problems; Boussinesg equations; Nonlinear wave equations; Schrodinger equations; Mathematical Medicine, Dynamics of tumor growth models.