Beauville p-Groups

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Abstract

Beauville surfaces are a class of rigid complex surfaces that have many nice geometric properties. A finite group giving rise to such a surface is called a *Beauville group*. What makes them so good to deal with is the fact that they can be described in purely group theoretical terms. A finite group G is a Beauville group if G is a 2-generator group and it has a pair of generating sets $\{x_1, y_1\}$ and $\{x_2, y_2\}$ such that $\Sigma(x_1, y_1) \cap \Sigma(x_2, y_1) = \{1\}$ where for $i = 1, 2$

$$\Sigma(x_i, y_i) = \bigcup_{g \in G} \Big(\langle x_i \rangle^g \cup \langle y_i \rangle^g \cup \langle x_i y_i \rangle^g \Big).$$

Catanese showed in 2000 that the abelian Beauville groups are those of the form $C_n \times C_n$ with $(n, 6) = 1$. After abelian groups, the most natural class of finite groups to consider are nilpotent groups. One can easily show that the study of nilpotent Beauville groups can be reduced to that of Beauville p-groups.

In this talk we survey a large collection of results on Beauville p-groups: from the earliest examples of Beauville p-groups to Beauville p-groups in the most known families of p-groups with a good behavior with respect to powers, such as regular p-groups, powerful p-groups, p-central p-groups etc.

We further focus on infinite families of Beauville p-groups arising from the quotients of infinite groups such as the free group, free product and triangle groups.