Bahçeşehir University, Istanbul, Turkey
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Institute Mathematics & Math. Modeling, Almaty, Kazakhstan

# "Analysis and Applied Mathematics"

Weekly Online Seminar

Seminar leaders:
Prof. Allaberen Ashyralyev (BAU, Istanbul),
Prof. Michael Ruzhansky (UGent, Ghent),
Prof. Makhmud Sadybekov (IMMM, Almaty)

Date: **Tuesday, December 27, 2022**
Time: 17.00-18.00 (Istanbul) = 15.00-16.00 (Ghent) = 20.00-21.00 (Almaty)

Zoom link: https://us02web.zoom.us/j/6678270445?pwd=SFNmQUIvT0tRaH-IDaVYrN3I5bzJVQT09, **Conference ID:** 667 827 0445, **Access code:** 1

Speaker:

**Prof. Dr. Michael V. Klibanov**
*University of North Carolina at Charlotte, USA*


Title: **Carleman Weighted Hilbert Spaces for Coefficient Inverse Problems**


Abstract: Coefficient Inverse Problems (CIPs) are both ill-posed and highly nonlinear. These two factors cause the non-convexity of conventional least squares cost functionals, which are constructed for numerical solutions of CIPs. The speaker with coauthors has developed a new approach to numerical solutions of CIPs, called convexification. The convexification constructs globally strictly convex cost functionals for a broad class of CIPs. This functional is defined on a bounded convex set of an arbitrary but fixed diameter in an appropriate Hilbert space, which we call Carleman Weighted Hilbert Space. The weight is the Carleman Weight Function, which is used in the Carleman estimate for a corresponding PDE operator. Uniqueness and existence of the minimizer of such a functional on that set is established. Convergence of minimizers to the true solution of the CIP is proven, provided that the noise in the data tends to zero. Many numerical examples, including ones with experimentally collected data, confirm the theory. Some of these results will be presented in my talk. Main contributers are (in the alphabetial order): Vo Khoa, Thuy Le and Loc Nguyen.


Biography:

Professor **Michael Victor Klibanov** is a distinguished expert in the field of Inverse Problems. He was born on February 2, 1950 in the city called Kuybyshev (currently Samara) in Russia. He has graduated from high school with distinction in 1967. In 1967–1972 he was a student of the Mathematics Faculty of Novosibirsk State University (NSU). NSU is one of the top Russian universities with the highest quality education in Mathematics. In 1972 Michael got his master's Diploma in Mathematics from NSU with distinction. In 1973–1977 he was working on his Ph.D.

under the supervision of Mikhail Mikhailovich Lavrentiev, one of the founders of the theory of Inverse and Ill-Posed Problems. Thus, the topic of Inverse Problems became the single topic of the research of Klibanov and he got outstanding results in Coefficient Inverse Problems (CIPs) for Partial Differential Equations (PDEs). He got his Ph.D. in Mathematics (Candidate of Physical and Mathematical Sciences) in 1977. In 1986 he was awarded Doctor of Science degree in Mathematics.

As of 1979, uniqueness and stability theorems for multidimensional CIPs with non-redundant data were proven only under very restrictive conditions imposed on the unknown coefficients. More precisely, people have assumed that this coefficient is either small in a certain sense or piecewise analytic, or something close to these. These were called local uniqueness theorems. However, a strong desire of specialists was to lift these assumptions and prove global uniqueness theorems, which would only assume that the unknown coefficient belongs to one of the commonly used function spaces, such as, e.g., $C^k$, $H^k$, etc. Proofs of global uniqueness theorems were a very substantial challenge at that time. Besides, even proofs of local uniqueness theorems were significantly dependent on specific PDEs. Michael has decided to break this "local" restriction. He has realized that to achieve this extremely challenging goal, it was absolutely necessary to move away from all existing at that time methods of investigations of those CIPs. He has figured out that as soon as a Carleman estimate can be derived for a Partial Differential Operator (PDO), then that idea can be applied almost immediately! On the other hand, since Carleman estimates are known for all three main types of PDOs of the second order, then the discovered method has turned out to be a very general one! But since it was known in 1980 from the book [12] that Carleman estimates can be derived for three main types of PDEs of the second order, parabolic, elliptic, and hyperbolic ones, then the resulting technique has turned out to be a very general one.

It turned out that Dr. Alexander L. Bukhgeim, who was also in the scientific school of Lavrentiev, got similar results simultaneously and independently of M. V. Klibanov. Thus, Bukhgeim and Klibanov have coauthored the first paper [3] on that method, which is currently commonly called "the Bukhgeim–Klibanov method" (BK). The first proofs were published in separate papers of Bukhgeim [2, Chapter 10, Section 3.8] and Klibanov [4]. The Bukhgeim–Klibanov method remains nowadays the most popular method allowing for proofs of global uniqueness and conditional stability results for multidimensional CIPs. As the result of this, in the past forty years, many people have used the rich ideas of BK for proofs of these results, see, e.g., references in [6, 9]. This is reflected in the fact that the paper [3] is currently (2022) cited more than 600 times on Google Scholar, In particular, the main follow up publications of Michael about applications of BK to proofs of those results (sometimes with coauthors) are three books [11] (2004), [1] (2012), [9] (2021) as well as many of his papers, see, e.g., [6] for his survey on the BK method as of 2013.

Michael never likes to work on topics which are more or less known how to handle. He always likes to mentally break "through the wall", whereas generously leaving to others to proceed further exploring his breakthrough ideas. Thus, as soon as the above breakthrough result has materialized the next question Michael has posed to himself was "What should be the main topic of my further research?" Observing the entire field of Inverse Problems, he eventually came to the conclusion that there is a very important topic which was left under-developed. This is the topic of globally convergent numerical methods for CIPs. Indeed, the topic of CIPs are undoubtedly very much applied ones. Therefore, reliable globally convergent numerical methods need to be developed for them. On the other hand, this line of developments was in its infancy stage.

Thus, Michael has beenworking for a long time on numerical methods for both CIPs and ill-posed Cauchy problems. In all cases Carleman estimates are the main tool of convergence analysis. This means that ideas of the first publication [3] were extended by Dr.

Klibanov in various ways to a number of numerical methods for these problems. We refer to the book [9] for a collection of those results. A much appreciated feature of Michael's research is that he likes to test his globally convergent numerical methods for CIPs not only on computationally simulated data, as it is commonly done, but on experimental data as well, see, e.g., [1, 9].

A very strong line of development of Michael is his work on a very tough inverse problem, the so-called "Travel Time Tomography Problem". Another name is "Inverse Kinematic Problem of Seismology". This problem has wide applications in Geophysics. The first works where this problem was solved in the 1-D case were works of German scientists G. Herglotz in 1905 and E. Wiechert and K. Zoeppritz in 1907. Since then, many attempts were made to extend these results to 2-D and 3-D cases.However, in all those publications only complete data were considered, i.e. it was assumed that sources and detectors run all over the entire boundary of the domain of interest. In particular, this implies that the data were redundant in the 3-D case. Globally convergent numerical methods were not developed for this problem. Klibanov was the first one who has considered in his breakthrough work [8] the case of incomplete data, which, in addition, are non-redundant in the 3-D case. In this publication he has constructed, for the first time, a globally convergent numerical method for this problem in 3-D. Furthermore, the linearized version of the travel time tomography problem is solved numerically [9, Chapter 12].

Finally, another line of results of Michael since his first publication [5] in 1987 is the topic of phaseless inverse scattering. In other words, this is the case when frequency dependent complex valued data for an inverse scattering problem contain only the absolute value of the scattered wave field, but do not have the phase information, see, e.g., [7, 10].

In summary, the Bukhgeim–Klibanov method [3] is currently one of the most general and powerful method in the entire field of Inverse Problems. It is very important that this method is applicable not only for proofs of uniqueness and stability results for Coefficient Inverse Problems but also to very powerful numerical methods. The convexification method of Klibanov is currently a single globally convergent numerical method for a broad variety of CIPs with non-overdetermined data. Professor Klibanov also got a number of other breakthrough results, the totality of which has made him a truly distinguished expert in the field of Inverse Problems.

## Eight breakthrough achievements of Professor Michael V. Klibanov

(1) Introduction of the tool of Carleman estimates in the field of Inverse Problems.
(2) Application of a combination of a Carleman estimate with the energy estimate to obtain Lipschitz stability estimate for general hyperbolic equations and inequalities.
(3) Convergence rates of the Quasi-ReversibilityMethod for ill-posed Cauchy problems for those linear PDEs, whose operators admit Carleman estimates.
(4) Globally convergent convexification method for ill-posed Cauchy problems for quasilinear PDEs.
(5) Globally convergent convexification method for computing the viscosity solution for the classical Hamilton–Jacobi equation.
(6) Globally convergent convexification method for a broad class of Coefficient Inverse Problems with nonredundant data.
(7) Particularly important cases of item (6) is two versions of the globally convergent convexification method for the classical travel time tomography problem in 3-D. Only incomplete non-overdetermined data are used. Thus, one can say that the problem, which was open since 1905 is solved!
(8) Uniqueness theorems and reconstruction procedures for phaseless inverse problems.

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