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Oleksandr Mikolaiovich Sharkovsky

Quick Info

Born
7 December 1936
Kiev, Ukranian SSR (now Kyiv, Ukraine)

Died
21 November 2022

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Summary
Oleksandr Sharkovsky was a Ukrainian mathematician who made contributions to the theory of discrete dynamical systems.

Biography

Oleksandr Mikolaiovich Sharkovsky's name often appears in different forms. His first name sometimes transliterated as Aleksandr or Alexander while his family name is often given as Sharkovskii or Sarkovskii. He attended school in Kiev (now often written as Kyiv) and in 1952, as an 8th grade pupil, he won the Kiev Mathematical Olympiad competition. He attended his local university of Kiev, the Kiev State University named for the Ukrainian poet and artist Taras Shevchenko. As a first year student, he produced his first original mathematics results on the asymptotics of algebraic curves. This proved to be the beginning of an outstanding research career. He graduated with a Master's Degree from the Department of Mechanics and Mathematics of Kiev State University in 1958 and worked on his Candidate's thesis at the Institute of Mathematics at the Ukrainian branch of the USSR Academy of Sciences. He was awarded his Candidate's Degree (equivalent to a Ph.D.) in 1961. He had already published a number of high quality papers (all in Russian), such as: *Necessary and sufficient conditions for convergence of one-dimensional iterative processes* (1960), *Rapidly converging iterative processes* (1961), *Solutions of a class of functional equations* (1961), and *The reducibility of a continuous function of a real variable and the structure of the stationary points of the corresponding iteration process* (1961).

In 1961 he was appointed to the Institute of Mathematics of the Academy of Sciences of the Ukraine in Kiev. He also taught at the University of Kiev from 1967 after defending his doctoral dissertation (equivalent in standard to the D.Sc. or the habilitation). He was made head of the Department of Differential Equations at the Institute of Mathematics at the Ukrainian branch of the USSR Academy of Sciences in 1974. He worked towards the creation of a Department of the Theory of Dynamical Systems in the Academy and, after it was founded, he became head of the department in 1986.

Sharkovsky's main areas of interest are the theory of dynamical systems, the theory of stability and the theory of oscillations. He also works in the theory of functional and functional differential equations, and the study of difference equations and their application. He is perhaps best known for an important theorem on periodic orbits in one-dimensional dynamical systems which he proved in 1964. He published this result, known today as Sharkovsky's Theorem, in the Russian paper *Co-existence of cycles of a continuous mapping of the line into itself* (1964). Although the result did not attract a great deal of interest at the time of its publication, during the 1970s other surprising results were proved which turned out to be special cases of Sharkovsky's theorem. The authors of [1] (in the translation [2]) write:-

> *Any contemporary monograph or textbook on the theory of dynamical systems can hardly be imagined without the Sharkovsky theorem. This theorem laid the foundation of a new branch in the theory of dynamical systems - combinatorial dynamics. The Sharkovsky theorem led to the appearance of numerous works in this direction, where one can often encounter terms such as the Sharkovsky theorem, Sharkovsky order, Sharkovsky space, Sharkovsky set, Sharkovsky stratification, and maximum period in the sense of Sharkovsky.*

Around the same time Sharkovsky published papers (some written in Russian, some in Ukrainian) such as: *Fixed points and the center of a continuous mapping of the line into itself* (1964), *On cycles and the structure of a continuous mapping* (1965), *On attracting and attracted sets* (1965), *Continuous mapping on a set of w-limit points* (1965), and *A classification of fixed points* (1965). The authors of [1] (in the translation [2]) write about his other contributions:-

> *Sharkovsky developed the foundations of the topological theory of one-dimensional dynamical systems, which is now one of the most efficient methods for studying various evolution problems. In particular, he investigated the relationship between the conditions of existence of periodic points with different periods and between the structure of the set of periodic points and the structure of attractors of trajectories. Furthermore, he investigated the topological structure of basins of attraction for various sets and deduced numerous criteria of simplicity or complexity of dynamical systems. Sharkovsky obtained fundamental results in the general theory of dynamical systems on arbitrary compact sets. In particular, he established the fundamental property, namely, incompressibility, of dynamical systems at attractors of trajectories. Sharkovsky also described types of global stability for almost all dynamical systems and established exact descriptive bounds for sets consisting of trajectories with different asymptotics. He showed that most of these upper bounds are attained even for one-dimensional dynamical systems. This fact implies that one-dimensional dynamical systems are, in a certain sense, as complex as dynamical systems in arbitrary spaces.*

Sharkovsky has, in collaboration with others, written a number of important monographs. The book, written with G P Pelyakh, *Introduction to the theory of functional equations* (Russian) was published in 1974. Alexandru Climescu writes in a review:-

> *The monograph is clearly written, but reading is sometimes weighed down by lengthy calculations.*

In 1986, in collaboration with Yu L Maistrenko and E Yu Romanenko, Sharkovsky published the Russian monograph *Difference equations and their applications*. The book contains four parts: (I) One-dimensional dynamical systems; (II) Difference equations with continuous time ; (III)

Differential-difference equations ; and (IV) Boundary-value problems for hyperbolic systems of partial differential equations. The authors write in the Preface:-

> *The aim of the present book is to acquaint the reader with some recently discovered and (at first sight) unusual properties of solutions to nonlinear difference equations. These properties enable us to use difference equations in order to model complicated oscillating processes (this can often be done in those cases when it is difficult to apply ordinary differential equations). Difference equations are also a useful tool in synergetics - an emerging science concerned with the study of ordered structures. The application of these equations opens up new approaches in solving one of the central problems of modern science - the problem of turbulence. Our presentation is mainly based on the modern theory of one-dimensional dynamical systems, interest in which has considerably grown recently.*

Marek Cezary Zdun writes in a review:-

> *Many of the results given in the book are new and have not been published elsewhere. This book is especially interesting for specialists in differential equations applying their results to some practical problems in the natural sciences and technology.*

An English translation was published in 1993.

Sharkovsky, in collaboration with S F Kolyada, A G Sivak and V V Fedorenko, published *Dynamics of one-dimensional mappings* in 1989. Feliks Przytycki writes in a review:-

> *The book is devoted to iterations of maps of the interval I. It contains introductory material as well as new advanced results. Many results are formulated without proofs or with only rough ideas of proofs. A pleasant side of the book is an abundance of examples.*

An English translation was published in 1997.

In 1978, Sharkovsky was elected as a corresponding member of the USSR Academy of Sciences. In 2006 he was elected as an Academician of the National Academy of Sciences of Ukraine (the name the Ukrainian Academy of Sciences took in 1994). In 1994, an international conference "Thirty years after Sharkovsky's theorem. New perspectives", was held in La Manga del Mar Menor, Murcia, Spain from 13 to 18 June. The papers show how much influence on the development of the theory of dynamical systems that Sharkovsky's results have had and continue to have. The publisher states:-

> *Since Professor A N Sharkovsky's landmark paper on the coexistence of periods for interval maps, several lines of research have been developed, opening applications of models to help understand a number of phenomena from a wide variety of fields, such as biology, economics, physics etc. The meeting served to summarize the progress made since Professor Sharkovsky's discovery, and to explore new directions.*

Thirty-seven papers were delivered at the conference and included in the proceedings, including the talk *Universal phenomena in some boundary value problems* by Sharkovsky himself. The Proceedings included an English translation of Sharkovsky's famous 1964 paper *Coexistence of cycles of a continuous map of the line into itself*.

As Head of the Institute of Mathematics of the National Academy of Sciences of Ukraine, Sharkovsky led a team which was awarded the 2010 State Prize of Ukraine in Science and Technology:-

> *For a series of scientific papers "Theory of Dynamical Systems: Methods and Applications".*

We end this biography by quoting from [2]:-

> *Sharkovsky has a rare talent to find and solve problems that seem specialized at first glance, but that acquire general scientific significance as time goes by. The results in the theory of dynamical systems obtained in the 1960s by Sharkovsky appeared to be very important when the necessity for a general investigation of essentially nonlinear processes arose. It turned out that many fundamental results and ideas of the contemporary theory of dynamical systems can be found in the early works of Sharkovsky. Some of his results were repeatedly reestablished and became real discoveries for specialists. Without any doubt, recent investigations of Sharkovsky into the mathematical aspects of nonlinear dynamics and, in particular, in the theory of turbulent oscillations will occupy with time an important place in various branches of science related to nonlinear processes.*

Other Mathematicians born in Ukraine
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