Jewish Problems

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Abstract

This is a special collection of problems that were given to select applicants during oral entrance exams to the math department of Moscow State University. These problems were designed to prevent Jewish people and other undesirables from getting a passing grade. Among problems that were used by the department to blackball unwanted candidate students, these problems are distinguished by having a simple solution that is difficult to find. Using problems with a simple solution protected the administration from extra complaints and appeals. This collection therefore has mathematical as well as historical value.

1 A personal story of Tanya Khovanova

In the summer of 1975, while I was in a Soviet math camp preparing to compete in the International Math Olympiad on behalf of the Soviet Union, my fellow team members and I were approached for help by Valera Senderov, a math teacher in one of Moscow's best special math schools.

The Mathematics Department of Moscow State University, the most prestigious mathematics school in Russia, was at that time actively trying to keep Jewish students (and other "undesirables") from enrolling in the department. One of the methods they used for doing this was to give the unwanted students a different set of problems on their oral exam. I was told that these problems were carefully designed to have elementary solutions (so that the Department could avoid scandals) that were nearly impossible to find. Any student who failed to answer could easily be rejected, so this system was an effective method of controlling admissions. These kinds of math problems were informally referred to as "Jewish" problems or "coffins". "Coffins" is the literal translation from Russian; they have also been called "killer" problems in English.

These problems and their solutions were, of course, kept secret, but Valera Senderov and his friends had managed to collect a list. In 1975, they approached us to solve these problems, so that they could train the Jewish and other students in these mathematical ideas. Our team of the best eight Soviet students, during the month we had the problems, solved only half of them. True, that we had other priorities, but this fact speaks to the difficulty of these problems.

Being young and impressionable, I was shaken by this whole situation. I had had no idea that such blatant discrimination had been going on. In addition to trying to solve them at the time, I kept these problems as my most valuable possession—I still have that teal notebook.

Later, I emigrated to the United States. When I started my own web page, one of the first things I did was to post some of the problems. People sent me more problems, and solutions to the ones I had. It turned out that not all of the coffins even had elementary solutions: some were intentionally ambiguous questions, some were just plain hard, some had impossible premises. This article is a selection from my collection; we picked out some choice problems that do contain interesting tricks or ideas.

<div align="right">Tanya Khovanova</div>

2 Introduction

Discrimination against Jewish people at the entrance exams to the most prestigious universities in the USSR is a documented fact, see A. Shen [1] and A. Vershik [4]. Alexander Shen in his article published 25 problems that were given to Jewish applicants. Later Ilan Vardi wrote solutions to all those problems, see I. Vardi's homepage [3]. These and other articles were published later in a book, "You Failed Your Math Test, Comrade Einstein: Adventures and Misadventures of Young Mathematicians, Or Test Your Skills in Almost Recreational Mathematics." [2].

In the present collection we have only one problem that overlaps with the problems that Vardi solved, and we give an easier and more surprising solution.

Now, after thirty years, these problems seem easier. Mostly, this is because the ideas of how to solve these problems have spread and are now a part of the standard set of ideas. Thirty years ago these problems were harder to solve and, in addition, the students were given these problems one after another until they failed one of them, at which point they were given a failing mark.

To give readers an opportunity to try and solve these problems we separated problems, key ideas, and solutions. Section 3 contains 21 problems. In the subsequent Section 4 we give hints or main ideas for the solutions. Finally, Section 5 contains solutions.

3 Problems

Problem 1

Solve the following inequality for positive x:

$$x(8\sqrt{1-x} + \sqrt{1+x}) \leq 11\sqrt{1+x} - 16\sqrt{1-x}. \tag{1}$$

Problem 2

Find all functions $F(x) : \mathbb{R} \to \mathbb{R}$ having the property that for any x_1 and x_2 the following inequality holds:

$$F(x_1) - F(x_2) \leq (x_1 - x_2)^2. \tag{2}$$

Problem 3

Given a triangle ABC, construct, using straightedge and compass, a point K on AB and a point M on BC, such that

$$AK = KM = MC. \tag{3}$$

Problem 4

Solve the following equation for real y:

$$2\sqrt[3]{2y-1} = y^3 + 1. \tag{4}$$

Problem 5

Solve the equation

$$\sin^7 x + \frac{1}{\sin^3 x} = \cos^7 x + \frac{1}{\cos^3 x}. \tag{5}$$

Problem 6

You are given a point M and an angle C in the plane. Using a ruler and a compass, draw a line through the point M which cuts off the angle C a triangle of
    a) a given perimeter p
    b) minimum perimeter.

Problem 7

There is a circle in the plane with a drawn diameter. Given a point, draw the perpendicular from the point to the diameter using only a straightedge. Assume the point is neither on the circle nor on the diameter line.

Problem 8

Given an equilateral triangle ABC and a point O inside it, with $\angle BOC = x$ and $\angle AOC = y$, find, in terms of x and y, the angles of the triangle with side lengths equal to AO, BO, and CO.

Problem 9

Two intersecting lines are given on a plane. Find the locus of points A such that the sum of the distances from A to each line is equal to a given number.

Problem 10

A quadrilateral is given in space, such that its edges are tangent to a sphere. Prove that all the points of tangency lie in one plane.

Problem 11

Prove that $\sin 10°$ is irrational.

Problem 12

Can you put six points on the plane, so that the distance between any two of them is an integer, and no three are collinear?

Problem 13

Is it possible to put an equilateral triangle onto a square grid so that all the vertices are in corners?

Problem 14

Reconstruct the quadrilateral $ABCD$ given the lengths of its sides (in order) and the length of the midline between the first and third sides (namely, all the segments drawn in Figure 1).

Problem 15

Find the quadrilateral with the largest area given the lengths of its four sides (in order).

Problem 16

You are given two parallel segments. Using a straightedge, divide one of them into six equal parts.
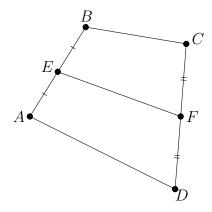
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Figure 1: Problem 14: Reconstruct this picture with ruler and compass, given only the lengths $|AB|, |BC|, |CD|, |DA|, |EF|$.

Problem 17

What's larger, $\log_2 3$ or $\log_3 5$?

Problem 18

How many digits does the number 125^{100} have?

Problem 19

Get rid of the radicals in the denominator of:

$$\frac{1}{\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c}}.$$

Problem 20

Construct (with ruler and compass) a square given one point from each side.

Problem 21

The graph of a monotonically increasing function is cut off with two horizontal lines. Find a point on the graph between intersections such that the sum of the two areas bounded by the lines, the graph and the vertical line through this point is minimum. See Figure 2.
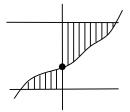
Figure 2: Problem 21: Given the graph, find the point that minimizes the depicted area.

4 Ideas

Idea for problem 1

Substitute

$$y = \frac{\sqrt{1-x}}{\sqrt{1+x}}. \tag{6}$$

Idea for problem 2

Use derivatives.

Idea for problem 3

Construct a triangle $A'B'C$ similar to ABC around points K' and M' so that the desired property holds for it.

Idea for problem 4

Define

$$x = (y^3 + 1)/2.$$

Idea for problem 5

$\cos^{2k+1} x - \sin^{2k+1} x = (\cos x - \sin x)P(\sin x \cos x)$, where P is a polynomial.

Idea for problem 6

Inscribe a circle into the angle C.
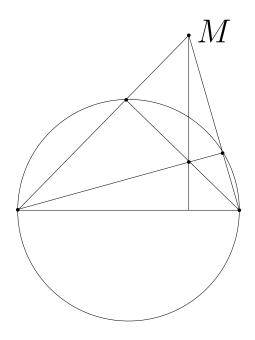
Idea for problem 7

See Figure 3.

Figure 3: Idea for Problem 7.

Idea for problem 8

Rotate the picture by $60°$ about A. Let the image of O be O'. Then the triangle COO' (or the triangle BOO' if we rotate in the other direction) will be the desired one. See Figure 4.

Idea for problem 9

In an isosceles triangle the sum of the distances from any point on the base to the two other sides is fixed.

Idea for problem 10

Use gravity.

Idea for problem 11

Express $\sin 10°$ through $\sin 30°$.

Idea for problem 12

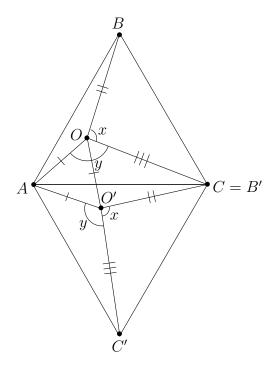A Pythagorean triangle generates three such points. Various reflections of it can increase the number of points.

Figure 4: Rotation for Problem 8.

Idea for problem 13

Use parity considerations or the fact that $\tan 60°$ is irrational.

Idea for problem 14

Consider the parallel translation of the sides AD and BC so that A' and B' are at E, shown in Figure 5. The midline will become the median of the new triangle $EC'D'$.

Idea for problem 15

It's the cyclic quadrilateral.

Idea for problem 16

The construction can be separated into two tasks: divide a segment into two equal parts, and divide a segment into six equal parts if six equal segments on a parallel line are given.
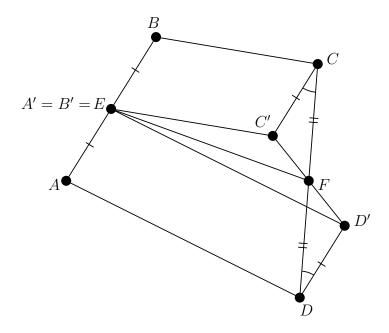
Figure 5: Parallel translations for Problem 14.

Idea for problem 17

Compare both values to $3/2$.

Idea for problem 18

Use the fact that $2^{10} = 1024$, which is close to 10^3.

Idea for problem 19

Use complex numbers.

Idea for problem 20

Cut two parallel lines at a given distance by another line at a given angle. The length of the cut out segment is fixed.

Idea for problem 21

The point on the middle line in between the two lines should work.

5   Solutions

Solution to problem 1

First, observe that for $x > 1$, the terms in (1) become undefined. Next, define y per the idea (6):

$$y = \frac{\sqrt{1-x}}{\sqrt{1+x}}.$$

Observe that for permissible values of x, we have

$$0 \le y \le 1,$$

and y is monotonically decreasing in x. Observe also that

$$x = \frac{1-y^2}{1+y^2}.$$

Given the above preliminaries, our inequality transforms as follows:

$$
\begin{aligned}
x(8\sqrt{1-x} + \sqrt{1+x}) &\le& 11\sqrt{1+x} - 16\sqrt{1-x} \\
x\left(8\frac{\sqrt{1-x}}{\sqrt{1+x}} + 1\right) &\le& 11 - 16\frac{\sqrt{1-x}}{\sqrt{1+x}} \\
\frac{1-y^2}{1+y^2}(8y+1) &\le& 11 - 16y \\
(1-y^2)(8y+1) &\le& (1+y^2)(11-16y) \\
-8y^3 - y^2 + 8y + 1 &\le& -16y^3 + 11y^2 - 16y + 11 \\
-8y^3 + 12y^2 - 24y + 10 &\ge& 0 \\
(2y-1)(-4y^2 + 4y - 10) &\ge& 0.
\end{aligned}
$$

Now, $-4y^2 + 4y - 10$ is always negative, so our monstrous inequality reduces, in the end, to the humble

$$2y - 1 \le 0.$$

Invoking the monotonicity of y in x and the fact that

$$\frac{1-(1/2)^2}{1+(1/2)^2} = \frac{3}{5},$$

our final answer is

$$\frac{3}{5} \le x \le 1.$$

Solution to problem 2

Inequality (2) implies

$$\frac{F(x_1) - F(x_2)}{|x_1 - x_2|} \le |x_1 - x_2|,$$

so the derivative of F at any point x_2 exists and is equal to zero. Therefore, by the fundamental theorem of calculus, the constant functions are exactly the functions with the desired property.
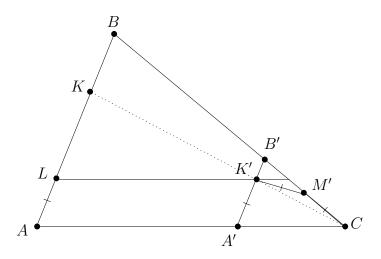
Solution to problem 3

Figure 6: Construction for Problem 3, in the order M', L, K', K.

Pick a point M' anywhere on BC. Construct L on AB such that

$$AL = CM'.$$

Now we want to parallel translate AL so that its image will satisfy (3) with CM'. To do this, construct through L a line parallel to AC, and on it construct the point K' such that

$$K'M' = CM'.$$

Construct through K' a line parallel to AB, and let it intersect the angle ACB at points A' and B'. Then the triangle $A'B'C$ and the points K' and M' satisfy (3), and $A'B'C$ is similar to ABC. Perform a homothety (also called a dilation) about C and you're done. Figure 6 illustrates.

Solution to problem 4

Define

$$x = (y^3 + 1)/2.$$

Then (4) becomes

$$y = (x^3 + 1)/2.$$

Since x and y are given by the same function g of each other, and g is monotonically increasing, we can conclude

$$x = y.$$

The equation is now a standard high-school cubic

$$y^3 - 2y + 1 = 0,$$

solvable by the method of guessing a small integer root and factoring. Indeed,

$$y = 1$$

is a root, and the factor $y^2 + y - 1$ yields two more roots

$$y = \frac{-1 \pm \sqrt{5}}{2}.$$

Solution to problem 5

First, rearrange (5) into

$$\frac{1}{\sin^3 x} - \frac{1}{\cos^3 x} = \cos^7 x - \sin^7 x$$

$$\frac{\cos^3 x - \sin^3 x}{\cos^3 x \sin^3 x} = \cos^7 x - \sin^7 x.$$

Now there are two cases:
Case 1, $\cos x - \sin x = 0$:
Then

$$x = \frac{\pi}{4} \quad \text{or} \quad x = \pi + \frac{\pi}{4}.$$

Case 2, otherwise:
We can cancel $\cos x - \sin x$, and get

$$\frac{\cos^2 x + \cos x \sin x + \sin^2 x}{\cos^3 x \sin^3 x} =$$
$$\cos^6 x + \cos^5 x \sin x + \cos^4 x \sin^2 x + \cos^3 x \sin^3 x +$$
$$\cos^2 x \sin^4 x + \cos x \sin^5 x + \sin^6 x. \qquad (7)$$

Substituting

$$t = \cos x \sin x,$$

we find that

$$\cos^4 x + \sin^4 x = (\cos^2 x + \sin^2 x)^2 - 2\cos^2 x \sin^2 x$$
$$= 1 - 2t^2$$

and that

$$\cos^6 x + \sin^6 x = (\cos^2 x + \sin^2 x)(\cos^4 x - \cos^2 x \sin^2 x + \sin^4 x)$$
$$= 1 - 3t^2$$

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so (7) reduces to

$$\frac{1+t}{t^3} = 1 - 3t^2 + t(1 - 2t^2) + t^2 + t^3 = 1 + t - 2t^2 - t^3$$

$$0 = -t^6 - 2t^5 + t^4 + t^3 - t - 1.$$

But

$$|t| = \left| \frac{\sin 2x}{2} \right| \leq \frac{1}{2},$$

so

$$| -t^6 - 2t^5 + t^4 + t^3 - t | \leq \frac{1}{64} + \frac{1}{16} + \frac{1}{16} + \frac{1}{8} + \frac{1}{2} < 1.$$

Therefore, there are no solutions in this case.

Solution to problem 6

a) The point M must be outside the angle opposite to C. Construct points A and B on the angle such that

$$\frac{p}{2} = CA = CB.$$

Then inscribe a circle into the angle tangent at those two points, and draw a tangent from M to that circle. The resulting triangle has the desired perimeter. This construction is illustrated in Figure 7.
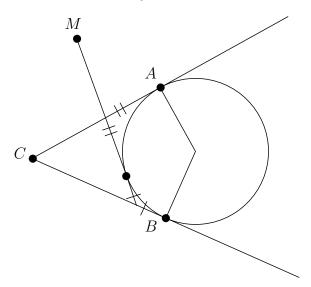

Figure 7: Construction for Problem 6a.

b) If M is outside the angle, the minimum perimeter is 0. If it is inside, then draw a circle through M away from C that is tangent to the angle, and take a

tangent to it as in part a. It is not hard to see from Figure 7 that the resulting triangle has minimum perimeter.

Solution to problem 7

The end points of the diameter and M form a triangle in which we need to draw a height, see Figure 3. The height passes through the orthocenter, which is easy to construct as the bases of the other two heights lie on the circle. If M is inside the circle, the construction is the same.

Solution to problem 8

The situation is represented in Figure 4. The points B, C, and O go to the points B', C', and O', respectively, under rotation about A by $60°$. The marked equalities follow by the properties of rotation. The desired triangle is then COO'. The angles in question are

$$
\begin{aligned}
O'OC &= y - 60°, \\
OO'C &= 300° - x - y, \\
OCO' &= 180° - (y - 60°) - (300° - x - y) \\
&= x - 60°.
\end{aligned}
$$

Solution to problem 9

In Figure 8, the sum of the areas of the two triangles ABD and CBD equals the area of the triangle ABC. Canceling the length of the side $BC(= AB)$, the sum of the heights from point D equals the height from point A (and the height from point C).

Therefore, the locus is a rectangle whose vertices are the four points that are each on one of the lines and at the desired distance from the other: Figure 9.

Solution to problem 10

Place masses at the vertices of the quadrilateral such that the center of mass of each edge falls onto a point of tangency. One way to do this is to place, at each vertex, a mass equal to 1 over the distance from that the vertex to each point of tangency of the edges that vertex is on (the two distances must be equal). Then the center of mass of the whole arrangement will have to be on each line connecting pairs of opposite points of tangency. Therefore, these lines intersect, and all four points of tangency are in one plane, as desired.

Solution to problem 11

Observe that

$$
\begin{aligned}
1/2 = \sin 30° &= 3 \sin 10° - 4 \sin^3 10° & (8) \\
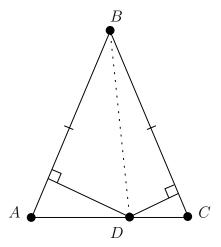0 &= 8 \sin^3 10° - 6 \sin 10° + 1, & (9)
\end{aligned}
$$

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Figure 8: One quadrant for Problem 9. $|BC| = |AB|$, so the sum of the heights from D is independent of the position of D on AC.
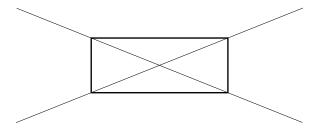

Figure 9: Answer to Problem 9: locus of points with fixed total distance from two given lines.

where (8) holds by repeated application of the sine and cosine angle sum formulae. Let

$$x = 2\sin 10°.$$

Then (9) reduces to

$$x^3 - 3x + 1 = 0.$$

All rational roots of this must be integers and divide the constant term. Since ± 1 do not work, all roots must be irrational.

Solution to problem 12

If we take a right triangle ABC with integer sides, the area will be rational. Therefore, the height onto the hypotenuse AC will be rational. This means that

if we reflect this triangle about the hypotenuse, we will have four points in the plane all at rational distances from each other.

Now, let the base of that height be at point D. Then, since the triangle ABC is similar to the triangle BDC, the distances AD and CD will both be rational. Therefore, reflecting the whole thing about the perpendicular bisector of AC yields two more points, so that all six are at rational distances from each other. All that's left now is to expand by a multiplicative factor that clears all the denominators.

Start with the 3, 4, 5 Pythagorean triangle, lay the hypotenuse along the x axis, with the center at the origin, perform the above construction, and scale by 10 to clear denominators. Six such points are:
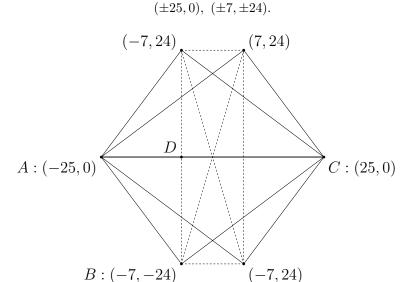
$$(\pm 25, 0), \ (\pm 7, \pm 24).$$

Figure 10: Answer to Problem 12. The six needed points have the given coordinates, and the points mentioned in the text are labeled. The solid lines have integral lengths by construction, being images under reflection of the initial Pythagorean triangle. The dashed lines have lengths that must be proven or verified to be integral.

Solution to problem 13

Suppose one of the vertices is at the origin. Suppose the coordinates of the two other vertices are (a, b) and (c, d). If all the numbers are divisible by two, we can reduce the triangle by half. Thus, we can assume that one of the numbers is not divisible by 2. Suppose a is odd.

If b is odd, then $a^2 + b^2$ is of the form $4k + 2$. As $a^2 + b^2 = c^2 + d^2$ we must conclude that both c and d are odd. But the square of the length of the

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third side is $(a - c)^2 + (b - d)^2$, which is divisible by 4. So the triangle can't be equilateral.

If b is even, then $a^2 + b^2$ is of the form $4k + 1$. As $a^2 + b^2 = c^2 + d^2$ we must conclude that c and d have different parity. Hence, $a - c$ and $b - d$ have the same parity and, correspondingly, the square of the length of the third side is again even. Contradiction.

For another solution, observe that the angle any segment between two grid vertices makes with either axis has to have a rational tangent. In addition, as

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta},$$

the tangent of the sum (or difference) of two angles with rational tangents is rational. Therefore any non-right angle given by grid points has rational tangent. Since $\tan 60° = \sqrt{3}$ is irrational, the equilateral triangle cannot have all three vertices on a square grid.

Solution to problem 14

Consider the parallel translation of the sides AD and BC so that A' and B' are at E, shown in Figure 5. Then the triangles $CC'F$ and $DD'F$ are equal, because they have two equal sides with an equal angle between them. Therefore, F is the median of the triangle $EC'D'$. We can reconstruct a triangle from the lengths of its two sides and the median between them,[1] which gives us the angle the original sides AD and BC made with each other.

The reconstruction of that triangle is equal to the triangle $AC''D$ in Figure 11, where AC'' is the parallel translation of BC so that B'' is at A. But we are given the distance between C and D and between C and C'', so we can find C by constructing the intersection of two circles. Constructing B is easy.

Solution to problem 15

Take the cyclic quadrilateral with the given sides and inscribe it into a circle. Consider the parts of the circle between the circumference and the quadrilateral as firmly attached to the sides. Consider a different quadrilateral with the given sides and the same parts attached to it shown in gray in Figure 12. The new figure will have the same perimeter as the perimeter of the circle, hence necessarily lesser area. Since the attached parts have fixed area, the quadrilateral must have lesser area as well.

Solution to problem 16

Given a segment and a parallel line we can always divide the segment into two equal parts. Take two points on the parallel line. Together with the ends of the

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[1] Because we can reconstruct the parallelogram whose sides are the given sides and one of whose diagonals has twice the length of the given median.
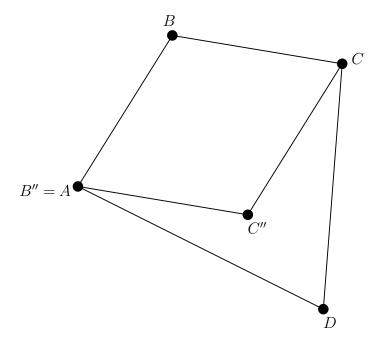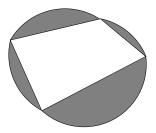
Figure 11: Completion of Problem 14.

Figure 12: A quadrilateral with attached parts, per the argument for Problem 15.

segment the points form a trapezoid. Continue the sides to form a triangle. The segment passing through the third vertex of the triangle and the intersection of the diagonals of the trapezoid divides both parallel segments into two equal parts.

Using the division method above, divide one of the segments into eight equal parts. Pick six of these parts consecutively. Then perform a homothety mapping their union onto the other segment. The center of the homothety is the intersection of the other sides of the trapezoid formed by the six segments and the target segment. See Figure 13.
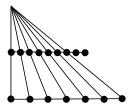

Figure 13: The homothety for Problem 16.

Solution to problem 17

We will compare $\log_2 3$ and $\log_3 5$ to $3/2$, both by the method of applying the same monotonic functions to both sides until we get something known.

First, compare $\log_2 3$ and $3/2$. The functions 2^x and x^2 are monotonic (at least for positive x), so we can use the composition of them to remove ugly pieces from our numbers:

$$\left(2^{\log_2 3}\right)^2 = 3^2 = 9 > 8 = 2^3 = \left(2^{3/2}\right)^2.$$

Second, compare $\log_3 5$ and $3/2$. The functions 3^x and x^2 are monotonic:

$$\left(3^{\log_3 5}\right)^2 = 5^2 = 25 < 27 = 3^3 = \left(3^{3/2}\right)^2.$$

Hence $\log_2 3 > \log_3 5$.

Solution to problem 18

Clearly, $125^{100} = 10^{300}/2^{300}$. Also, $2^{300} = 1024^{30} = 10^{90} \times 1.024^{30}$. So

$$125^{100} = 10^{210}/1.024^{30}.$$

Let's estimate 1.024^{30}. For any number slightly bigger than 1 we can use the binomial formula: $(1 + x)^{30} = 1 + 30x + 435x^2 + \binom{30}{3}x^3 + \ldots$. In our case $x < 1/40$. We can estimate each term after 1 as 0.75, 0.27, and 0.06. We see that they are getting smaller very fast. So we know that $1 < 1.024^{30} < 10$, which gives us 210 digits for 125^{100}.

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Solution to problem 19

Let $x = \sqrt[3]{a}$, $y = \sqrt[3]{b}$, and $z = \sqrt[3]{c}$. Consider three numbers: 1, w, w^2, where w is a primitive cube root of 1. Consider the nine terms of the form $x + w^i y + w^j z$, where i and j are 0, 1, or 2. If we multiply them we get an expression with all real coefficients, because it is self-conjugate. In addition, replacing y with wy doesn't change the product, so the powers of y present in the result have to be divisible by 3. Analogously, the powers of z have to be divisible by 3. Consequently, as the product is homogeneous, the powers of x have to be divisible by 3.

Thus the resulting product is a polynomial of a, b, and c. Explicitly, if we multiply the denominator and numerator by $(x + wy + w^2 z)(x + w^2 y + wz)(x + y + wz)(x + y + w^2 z)(x + wy + z)(x + w^2 y + z)(x + wy + wz)(x + w^2 y + w^2 z)$, the denominator becomes $(a + b + c)^3 - 27abc$.

We might try to simplify the calculation by initially multiplying the numerator and denominator by $(x + wy + w^2 z)(x + w^2 y + wz) = x^2 + y^2 - yz + z^2 - x(y + z)$. We get

$$\frac{1}{\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c}} = \frac{\sqrt[3]{a}^2 + \sqrt[3]{b}^2 + \sqrt[3]{c}^2 - \sqrt[3]{ab} - \sqrt[3]{ac} - \sqrt[3]{bc}}{a + b + c - 3\sqrt[3]{abc}}.$$

Now the denominator is of the form $s - \sqrt[3]{t}$ and has only one radical. To get rid of it we can multiply the denominator and the numerator by $s^2 + s\sqrt[3]{t} + \sqrt[3]{t}^2$. The denominator becomes $s^3 - t = (a + b + c)^3 - 27abc$.

Solution to problem 20

Suppose A, B, C and D denote the points on the sides of the square, in order. Connect A and C. Draw a perpendicular from B to the segment AC. Let point D' lie on the perpendicular given that the distance BD' is the same as the length of AC. Then D' belongs to the same side (or extension of the side) of the square as D. See Figure 14. Once we know from DD' how the square is oriented, the rest is easy.
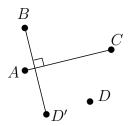

Figure 14: Problem 20: Constructing a square from four given points.

Solution to problem 21

Let A be the point on the function that lies on the middle line between and parallel to the two lines. If we move the cut off point to the right, then the area under the function increases faster than the area over the function decreases. Moving to the left is symmetric.

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