Boğaziçi Math Seminar

Almost Periodicity and Fourier Quasicrystals

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Abstract:

Bohr (resp. Besicovich) developed almost periodic function theory to study $\zeta(x+iy)$ for $x>1$ (resp. $x>1/2$). Levin showed that divisors of bounded holomorphic $f(x+iy)$ AP in x for $a<y<b$ are AP multisets. Fourier transforms $F(\mu)$ of measures μ on $\mathbb{R}^n$ identified with AP multisets in $\mathbb{R}^n$ are pure point distributions on $\mathbb{R}^n$ whose properties describe the type of μ

Poisson Measure: $F(\mu)$ is a Radon measure
Crystalline Measure: support $F(\mu)$ is discrete
Fourier Quasicrystal: CM and variation $|F(\mu)|$ is tempered

PM $\rightarrow$ CM $\rightarrow$ FQ and FQ $\rightarrow$ Bohr AP. Meyer model sets, which describe physical quasicrystals synthesized by Schectman and colleagues and independently hypothesized by Levine and Steinhardt, are not PM, and Lagarias proved they are Besicovitch but not Bohr AP. Favorov constructed a CM that is not a FQ. We apply topological dynamcs and harmonic analysis to relate these AP objects to compactifications of $\mathbb{R}^n$. Kurasov and Sarnak constructed the first nontrivial FQ on $\mathbb{R}$. They are divisors of real-rooted trigonometric polynomials. Olevsky and Ulanovsky proved that all FQ on $\mathbb{R}$ arise this way. We construct FQ on $\mathbb{R}^n$ using a formula derived in a 2022 paper on Bohr AP sets of toral type, and in a recently resubmitted paper derived sufficient conditions for divisors of trigonometric maps to be FQ on $\mathbb{R}^n$ using Grothendieck residues and toric geometry.

Date : Wednesday, October 9, 2024
Time: 13:30
Place: TB 130, Boğaziçi University