Bahçeşehir University, Istanbul, Türkiye
Analysis & PDE Center, Ghent University, Ghent, Belgium
Institute Mathematics & Math. Modeling, Almaty, Kazakhstan

"Analysis and Applied Mathematics"

Weekly Online Seminar

Seminar leaders:
Prof. Allaberen Ashyralyev (BAU, Istanbul),
Prof. Michael Ruzhansky (UGent, Ghent),
Prof. Makhmud Sadybekov (IMMM, Almaty)

Date: **Tuesday, November 26, 2024**
Time: 14.00-15.00 (Istanbul) = 12.00-13.00 (Ghent) = 16.00-17.00 (Almaty)

Zoom link: https://us02web.zoom.us/j/6678270445?pwd=SFNmQUIvT0tRaHlDa-VYrN3l5bzJVQT09, **Conference ID:** 667 827 0445, **Access code:** 1

Speaker:

Assoc. Prof. Dr. Maksim V. Kukushkin
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Title: **On the infinitesimalness of the summation order in the Abell-Lidskii sense for the trace class**

Abstract: In the recent century the problem of root vectors system completeness related to non-selfadjoint operators is undergone a serious attention by such mathematicians as Markus A.S. [16], [17], Lidskii V.B. [14], Krein M.G. [7], Katsnelson V.E. [6], Matsaev V.I.[18], Agranovich M.S. [2] and others. In consequence, there appeared a fundamental concept in the framework of abstract spectral theory including propositions on summation of spectral decompositions (series on root vectors) in a generalized sense such as Abel-Lidskii, Riesz, Bari, senses [2],[5].

The problem of decreasing of the summation order in the Abell-Lidskii sense was formulated by Lidskii V.B. 1962 [15] for a case corresponding to the selfadjoint elliptic operator perturbed by a non-selfadjoint operator. More generally, the problem was considered by Katsnelson V.E. 1967 [3] for perturbations of a positive selfadjoint operator under the strong subordination condition [19]. In 1994, Agaranovich M.S. proved that the summation order can be decreased to some positive number in the case corresponding to an operator with the numerical range of values containing in the domain of the parabolic type [2] (what is an essential restriction in comparison with the sectorial condition). However, a problem on the lower bound of the summation order has not been still solved.

In this report we will show that the summation order in the Abell-Lidskii sense can be decreased to an arbitrary small positive value in the case corresponding to the sectorial operator belonging to the trace class. In addition, we construct a qualitative theory of summation in the Abell-Lidkii sense and produce relevant applications in the theory of pseudo-differential operators.

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Biography:

Maksim V. Kukushkin has obtained a degree in Mathematics in 2002 and PhD degree in Mathematics in 2016, last of them from Ministry of Education and Science of Russian Federation (Russia). After completion of his studies, he had a position of Associate Professor in Saint-Petersburg State University of Aerospace Instrumentation, Emperor Alexander I Saint Petersburg State Transport University, Kabardino-Balkarian Scientific Center, RAS, Moscow State University of Civil Engineering. His topics of research deal mostly with the spectral theory of non-selfadjoint operators, the applications to the semigroup theory, as well as to evolution equations in the abstract Hilbert space.